Stanford PhD algebra qualifying exam material
May 6, 2016

The following sources are references for the assumed background listed below, as well as for much (but not all) of the further material that is covered in the courses:


The courses 210A and 210B cover the material below that is not within the “assumed background” list. These courses also cover additional material (chosen by the specific instructor, so varying from year to year) to fulfill the additional educational purpose of being graduate-level introductory courses in algebra.

Assumed background.

The material listed within the following 3 core topics constitute a mixture of concepts that we expect you have learned during your undergraduate studies or can acquire on your own from the indicated references before arriving at Stanford. Make sure to understand examples and to work out exercises (which [DF] provides in abundance for group theory, and [L] provides in abundance for field theory).

Group theory

Normal subgroups, simple groups, and Jordan–Holder theorem [L, I.3], group actions on sets (including symmetric and alternating groups, and simplicity of $A_n$ ($n \geq 5$)) [L, I.5], Sylow theorems and applications (including solvability of $p$-groups) [L, I.6], semi-direct products [DF, §5.5], free groups and group presentations [DF, §6.3].

Linear algebra

Characteristic polynomials, eigenvalues and eigenvectors, dual space, trace and determinant, diagonalization. ([DF, Ch. 11], [L, Ch. XIII])

Field theory

Degree of a field extension, splitting field, finite fields, theorem of the primitive element, field embeddings and automorphisms, Galois theory, solvability of equations by radicals. ([DF, Ch. 13–14], [L, Ch. V–VI])

210A Course material

Basic commutative rings

Principal ideal domains, unique factorization domains, polynomial and power series rings, prime and maximal ideals, Chinese Remainder Theorem for concomaximal ideals. Local rings and localization, Nakayama’s lemma, Noetherian rings, Hilbert basis theorem. ([DF, Ch. 7–9, Ch. 15], [L, Ch. II, Ch. IV])

Modules and homological methods

Finitely generated modules over PIDs, applications (Jordan and rational canonical form); chain conditions (esp. Noetherian modules), tensor product and Hom, flatness, categories and functors, projective and injective resolutions, Tor and Ext. ([DF, Ch. 10, Ch. 12, Ch. 17], [L, Ch. III, Ch. X, Ch. XVI, Ch. XX])

Advanced linear algebra
Minimal polynomial, Jordan and rational canonical forms, Jordan decomposition for endomorphisms and automorphisms, multilinear algebra, tensor algebra (tensor powers, symmetric and exterior powers), bilinear forms (symmetric, alternating), quadratic forms, spectral theorem for finite-dimensional $\mathbb{R}$-vector spaces, signature ([DF, §11.5, Ch. 12], [L, Ch. XIII–XIV, XV.1–XV.4, XV.8])

210B Course material

Advanced Field theory

Algebraic extensions and algebraic closure, separable and inseparable extensions, norm and trace, transcendence degree, finitely generated extensions of a field. ([DF, §14.9], [L, Ch. V–VI, VIII.1])

Representation theory

Group algebra and group representations over a field, irreducible representations of finite groups, Schur’s lemma, Maschke’s theorem, characters (only in characteristic 0), Schur orthogonality, character tables, induced representations, Frobenius reciprocity. ([DF, §18.1, §18.3, §19.3], [L, Ch. XVIII])

Commutative algebra

Artin rings, integrality, going-up and going-down theorems (for integral extensions), Dedekind domains and discrete valuation rings, Artin–Rees Lemma and completion of modules and rings, $p$-adic integers and power series rings, flatness of completion.

Nullstellensatz, affine algebraic sets and irreducible components (over an algebraically closed field), relation of Dedekind domains to smoothness for plane algebraic curves, Spec($A$) and Zariski topology, Noether normalization, dimension theory and normalization for finitely generated algebras over a field. ([DF, Ch. 16], [L, Ch. II, IV, VII, IX, X])