Abstract: This is a talk on counting solutions of non-linear elliptic PDEs. After presenting the basic idea, I will explain, from two completely different perspectives, how the search for simple examples leads—rather surprisingly—to considering holomorphic maps into Calabi-Yau 3-folds $X$. Such maps are counted by the Gromov-Witten invariants of $X$, which are an infinite set of rational numbers. In 1998, physicists R. Gopakumar and C. Vafa conjectured that these Gromov-Witten invariants have a hidden structure: they are obtained, by a specific transform, from a set of more fundamental “BPS numbers,” which are integers. The talk will conclude with a pictorial proof of the GV conjecture (joint work with E. Ionel) based on the idea of using deformations of almost complex structures to count the contributions of “clusters” of curves.