A $p$-adic approach to Hilbert’s 12th problem

Samit Dasgupta
UCSC

Abstract

It is classical that the square root of any integer can be written as a linear combination of roots of unity. A generalization of this fact is the ”Kronecker-Weber Theorem”, which states that any complex number that generates an abelian Galois extension of the field of rational numbers $\mathbb{Q}$ can also be written as such a linear combination. The roots of unity may by viewed as the special values of the analytic function $e(x) = \exp(2\pi ix)$, where $x$ is taken to be a rational number. Broadly speaking, Hilbert’s 12th problem is to find an analogous result when $\mathbb{Q}$ is replaced by a general algebraic number field $F$, and in particular to find the analytic functions that play the role of $e(x)$ in this general setting.

Hilbert’s 12th problem has been solved in the case where $F$ is an imaginary quadratic field, with the role of $e(x)$ being played by certain modular forms. All other cases are, generally speaking, unresolved. In this talk I will discuss the case where $F$ is a real quadratic field, and more generally, a totally real field. I will describe relevant conjectures of Stark and Gross, as well as current work using a $p$-adic approach and methods of Shintani. A proof of these conjectures would arguably provide a positive resolution of Hilbert’s 12th problem in these cases.

Thursday, October 1
4:15 p.m.
Bldg. 380, Room 380-W.

http://math.stanford.edu/coll/0910/
Fair allocations to random points, using the stable marriage algorithm, Riemann mapping theorem, and Newtonian gravity

Yuval Peres
Microsoft

Abstract
Given an infinite collection of points in the plane (a point process), how do we allocate the same area to each point in a decentralized, shift-invariant way? Different approaches to this problem have connections with probability, combinatorics, ergodic theory, the Riemann mapping theorem, and Newtonian gravity (in higher dimensions); see the gallery at http://www.math.ucdavis.edu/romik/home/Allocations.html but there is lots of room for new creative ideas.

This exposition of the topic will be based on work of the speaker as well as C. Hoffman, A. Holroyd, S. Chatterjee, R. Peled, D. Romik and M. Krikun.

Thursday, October 15
4:15 p.m.
Bldg. 380, Room 380-W.

http://math.stanford.edu/coll/0910/
Stanford Department of Mathematics
Colloquium

Thursday, October 29
4:15 p.m.
Bldg. 380, Room 380-W.

SPINORS AND THE DYNAMICS OF VECTOR FIELDS IN
DIMENSION 3.

Clifford H. Taubes
Harvard

Abstract
As it turns out, the Dirac equation can be used to say something about the integral curves
of certain sorts of vector fields on 3-dimensional manifolds. How this comes about is a long
and detailed story. Even so, the plot outline is accessible to all–expert status not required.

http://math.stanford.edu/coll/0910/
Fukaya categories and bordered Heegard-Floer homology

Denis Auroux
Berkeley/MIT

Abstract
An important feature of many invariants in low-dimensional topology is their functoriality with respect to decompositions of manifolds into smaller pieces. In the case of Heegaard-Floer homology (one of the most powerful 3-manifold invariants available), such a property was recently established by Robert Lipshitz, Peter Ozsvath and Dylan Thurston, whose "bordered Heegaard-Floer homology" associates to a 3-manifold with boundary $F$ a pair of modules over a certain algebra $A(F)$.

After briefly reviewing the construction of bordered Heegaard-Floer homology, we will attempt to re-interpret it in terms of the symplectic topology of symmetric products. More specifically, we will explain how to understand the algebra $A(F)$ associated to a surface $F$ in terms of a (relative) Fukaya category of the symmetric product of $F$; if time permits we will outline the corresponding description of the module $CFA(Y)$ associated to a bordered 3-manifold.

Thursday, November 12
4:15 p.m.
Bldg. 380, Room 380-W.

http://math.stanford.edu/coll/0910/
Categorification of the Fourier transform

Jacob Lurie
Harvard

Abstract

In this talk, we will describe the process of "categorification": that is, taking a known statement about concrete objects (like sets), and looking for a generalization (or analogue) in a more abstract context (like category theory). We will give some specific examples, beginning with the classical theory of the Fourier transform, and (if time permits) explain how the "categorification" of this theory is relevant to the geometric Langlands program.
Non-linear problems involving integral diffusions

Luis Caffarelli
UT Austin

Abstract
This will be a survey on recent work involving the regularity theory for the non-local equivalent to the Calculus of Variations and fully non-linear equations (DeGiorgi, Krylov Safonov, Evans).

http://math.stanford.edu/coll/0910/
Function theory on symplectic manifolds

Leonid Polterovich
Tel Aviv/MSRI

Abstract

It has been recently observed that function spaces associated to a symplectic manifold exhibit unexpected properties and surprising structures, giving rise to new tools and intuition in symplectic topology. In the talk I shall discuss these developments, as well as links to other subjects such as group theory, Lie algebras, and the quantum-classical correspondence. The talk is based on a series of joint works with Michael Entov.
BORDERED HEEGAARD FLOER HOMOLOGY

Peter Ozsvath
MSRI/Columbia

Abstract

Heegaard Floer homology is an invariant for three- and four-manifolds defined, in joint work with Zoltan Szabo, using methods from symplectic geometry. After reviewing properties and some applications of these invariants I will describe an extension, bordered Heegaard Floer homology, to three-manifolds with boundary. I will discuss the algebraic structure of this extension, and give its relationship with the earlier invariants for closed three-manifolds. This is joint work with Robert Lipshitz and Dylan Thurston.

http://math.stanford.edu/coll/0910/
SYMPLECTIC EMBEDDINGS AND CONTINUED FRACTIONS

Dusa McDuff
MSRI/Columbia

Abstract
As has been known since the time of Gromov’s Nonsqueezing Theorem, symplectic embedding questions lie at the heart of symplectic geometry. This talk will discuss some recent joint work with Schlenk concerning the question of when a 4-dimensional ellipsoid can be symplectically embedded in a ball. This problem turns out to have unexpected relations to the properties of continued fractions, counts of lattice points in triangles, and exceptional curves in blow ups of the complex projective plane.
The geometry and topology of arithmetic hyperbolic 3-manifolds

Alan Reid
UT Austin

Abstract

Arithmetic hyperbolic 3-manifolds form an interesting and important class of hyperbolic 3-manifolds; in part because of their connections with number theory and the theory of automorphic forms. This talk will survey recent work in understanding the geometry and topology of finite sheeted covering spaces of these manifolds in the context of various open problems about the topology of covers of closed hyperbolic 3-manifolds.
SYMPLECTIC HOMOGENIZATION AND APPLICATIONS

CLAUDE VITERBO (Polytechnique)

Abstract

The question of the convergence of a sequence of Hamiltonians $H(kq, p)$ for $(q, p)$ on the torus is a problem that has appeared in different contexts: Dynamical systems, PDE, etc. We here explain how a symplectic topology point of view can yield a solution which, among other advantages, does not require convexity. If time permits, we shall try to explain how this is related to a (partial) extension of Aubry-Mather theory to the non-convex framework.
Complex dynamics and adelic potential theory

Matt Baker
Georgia Tech

Abstract

Using tools from number theory and complex analysis, Laura DeMarco and I have recently proved the following theorem: for any fixed complex numbers $a$ and $b$, the set of complex numbers $c$ for which both $a$ and $b$ both have finite orbit under the map $z \mapsto z^2 + c$ is infinite if and only if $a^2 = b^2$. I will explain the motivation for this result and give an outline of the proof. The main arithmetic ingredient in the proof is an adelic equidistribution theorem for preperiodic points over product formula fields, with non-archimedean Berkovich spaces playing an essential role.
Marching to a different drummer

David Mumford
Brown/Berkeley

Abstract

Mathematics is conventionally seen as the activity of proving theorems for which the paradigm is Euclid’s Elements. It is rare that you can replay history but, in this case, we can study how Mathematics developed in a largely indigenous way in India. I will present a Western mathematician’s view of their mode of pursuing mathematics. I want to show a little of how deeply they penetrated into algebra and analysis prior to the 16th century tidal wave of foreign invasions.
Packing space with regular tetrahedra

Jeff Lagarias
University of Michigan

Abstract

The problem of the densest packing of space by congruent regular tetrahedra has a long history, starting with Aristotle’s assertion that regular tetrahedra fill space, and continuing through its appearance in Hilbert’s 18th Problem. This talk describes its history and many recent results obtained on this problem, including contributions from physicists, chemists, and materials scientists. The current record for packing density (as of January 5, 2010) is held by my graduate student Elizabeth Chen.
Variation with $p$ of the number of solutions mod $p$

of polynomial equations

J-P. Serre
Collège de France

Abstract

Let $f = (f_1, f_2, \ldots)$ be a family of polynomials in several variables, with coefficients in $\mathbb{Z}$. If $p$ is a prime number, let $N(f; p)$ be the number of solutions mod $p$ of the system of equations $f(x) \equiv 0 \mod p$. We shall discuss the way $N(f; p)$ varies with $p$: closed formulae, computability, size, congruence properties, relations with topology, etc.
Random hyperbolic curves and geometry of the moduli space

Maryam Mirzakhani
Stanford

Abstract

In this talk, we describe the relationship between the geometry of hyperbolic surfaces and properties of the moduli space of such surfaces. In particular, we discuss the behavior of lengths of simple closed geodesics on a random hyperbolic surface of genus $g$ with respect to the Weil-Peterson measure as $g \to \infty$. 

Thursday, May 27
4:15 p.m.
Bldg. 380, Room 380-W.

http://math.stanford.edu/coll/0910/