Abstract

I will give a gentle introduction to equivariant vector bundles on toric varieties, an emerging research area in which a combination of combinatorial and geometric methods can be used to test difficult general questions about algebraic vector bundles. For instance, does every complete algebraic variety have a nontrivial vector bundle on it?
Stanford Department of Mathematics
Colloquium

RICCI FLOW IN HIGHER DIMENSION AND THE SPHERE THEOREM

SIMON BRENDLE
Stanford

Abstract

We describe recent joint work with Richard Schoen on the Ricci flow in higher dimensions. We discuss various new curvature conditions, which are based on the notion of positive isotropic curvature (PIC) and are preserved by the Ricci flow. Using these ideas, we obtain a new convergence result for a class of manifolds that includes all manifolds with 1/4-pinched sectional curvatures. As a corollary, we give an affirmative answer to a question posed by H. Rauch in 1951.

Thursday, November 1
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
THE STORY OF THE SPARSEST CUT PROBLEM

ASSAF NAOR
New York University

Abstract

In the past decade methods from Riemannian geometry and Banach space theory have become a central tool in the design and analysis of approximation algorithms for a wide range of NP hard problems. In the reverse direction, problems and methods from theoretical computer science have recently led to solutions of long standing problems in metric geometry. This talk will illustrate the connection between these fields through the example of the Sparsest Cut problem. This problem asks for a polynomial time algorithm which computes the Cheeger constant of a given finite graph. The Sparsest Cut problem is known to be NP hard, but it is of great interest to devise efficient algorithms which compute the Cheeger constant up to a small multiplicative error. We will show how a simple linear programming formulation of this problem leads to a question on bi-Lipschitz embeddings of finite metric spaces into $L_1$, which has been solved by Bourgain in 1986. We will then proceed to study a quadratic variant of this approach which leads to the best known approximation algorithm for the Sparsest Cut problem. The investigation of this “semidefinite relaxation” leads to delicate questions in metric geometry and isoperimetry, in which the geometry of the Heisenberg group plays an unexpected role.

Thursday, November 8
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
PLATO’S CAVE: WHAT WE STILL DON’T KNOW ABOUT THE PROJECTIONS IN PROJECTIVE GEOMETRY

DAVID EISENBUD
UC Berkeley

Abstract

Varieties can be studied by projecting them until they are hypersurfaces. How much is lost? I’ll talk about some of the history, about what is known, and about the puzzles that remain. I’ll also mention some recent joint results of mine with Roya Beheshti.

Thursday, November 15
4:15 p.m.
Room 380-W

http://math.stanford.edu/coll/0708/
Origins of Positivity in Algebraic Geometry

William Fulton
University of Michigan

Abstract

Positivity can be traced back to the beginnings of algebraic geometry, from Bezout’s theorem about intersections of projective varieties, to theorems of Bertini and Lefschetz, describing how hyperplane sections resemble the varieties they intersect. New notions of positivity were discovered in the 1970’s, coming from relations between singularities and characteristic classes. The theorems and applications that came from this can be found in recent books of Lazarsfeld. This talk will sketch the story of how it happened.
INTEGRATED HARNACK INEQUALITIES

Bruce Driver
UCSD

Abstract

If $p_t(x)$ is the standard heat kernel on $\mathbb{R}^d$, then a simple computation shows, for all $y \in \mathbb{R}^d$ and $1 \leq p < \infty$, that

$$
\left( \int_{\mathbb{R}^d} \left[ \frac{p_t(x - y)}{p_t(x)} \right]^p p_t(x) \, dx \right)^{\frac{1}{p}} = \exp \left( -\frac{(p - 1)}{2t} |y|^2 \right).
$$

In this talk we will explain what happens to this equality when $\mathbb{R}^d$ is replaced by a unimodular Lie group. If time permits, I will explain how the results of this talk can be viewed as the first step towards proving parabolic regularity in infinite dimensional contexts. (This is joint work with Masha Gordina.)

Thursday, November 29
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
STABILITY OF STRONG VISCOUS SHOCK LAYERS IN AN IDEAL GAS

KEVIN ZUMBRUN
Indiana

Abstract
By a combination of asymptotic ODE estimates and numerical Evans-function computations, we examine the spectral stability of shock-wave solutions of the compressible Navier–Stokes equations with ideal gas equation of state, for arbitrary strength shock waves. Our main results are that, in appropriately rescaled coordinates, the Evans function associated with the linearized operator L about the wave, an analytic function analogous to the characteristic polynomial whose zeros correspond to eigenvalues of L, (i) converges in the strong shock limit to the Evans function for a limiting shock profile of the same equations, for which internal energy vanishes at one endstate; and (ii) has no unstable (positive real part) zeros outside a uniform ball. Thus, the rescaled eigenvalue ODE for the set of all shock waves, augmented with the (nonphysical) limiting case, form a compact family of boundary-value problems that can be conveniently investigated numerically. An extensive numerical study then yields unconditional stability, independent of amplitude for a range of parameter values including all common gases.

Besides its physical interest, we believe that this analysis has interest as an example where it is possible to carry out a rigorous global stability analysis by numerical techniques, the obvious obstacle being the need to treat an unbounded parameter range using finitely many operations.

Thursday, January 10
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
In this talk I will discuss the question of global regularity of the three-dimensional Navier-Stokes equations and other related equations in fluid dynamics. I will emphasize the mathematical as well as the physical difficulties in achieving such global regularity result. Moreover, I will discuss the effect of rotation on “regularizing” three-dimensional flows.
Stanford Department of Mathematics
Colloquium

ADIABATIC LIMITS, SURGERY AND OPERATORS

RICHARD MELROSE
MIT

Abstract

Starting from the semiclassical limit in quantum mechanics I will describe the metric adiabatic limit for a fibration and certain surgery limits, their associated configuration spaces and Lie algebras (or groupoids) and related analytic results. As time permits I will also describe similar constructions for singular fibrations of Morse type.

Thursday, January 31
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
Abstract

I will discuss some applications of algebraic topology to quantum field theory and string theory. Recent examples include the Wess-Zumino-Witten term in quantum chromodynamics and orientifolds in string theory. Topology accesses parts of these theories which are independent of scale, as we explain in these examples.
Groups of automorphisms of free groups act in interesting ways on a variety of beautiful geometric objects. I will describe some of these objects and explain how they are used to establish algebraic properties of the automorphism groups. In contrast, I will also describe rigidity phenomena for these groups which in particular restrict the types of objects on which they can act in any non-trivial way.
ON THE BOREL CONJECTURE AND RELATED TOPICS

WOLFGANG LÜCK
Universität Münster

Abstract

The Borel Conjecture predicts for two closed aspherical manifolds $M$ and $N$ that they are homeomorphic if and only if their fundamental groups agree and that in this case every homotopy equivalence is homotopic to a homeomorphism. This may be viewed as the topological version of Mostow rigidity. It is more or less closely related to the Farrell-Jones Conjecture about the algebraic $K$- and $L$-theory of group rings and the Baum Connes Conjecture about the topological $K$-theory of reduced group $C^*$-algebras. We present the recent work together with Bartels, where we prove the Farrell-Jones Conjecture and hence the Borel Conjecture in dimension greater or equal to 5 for a large class of groups which includes word-hyperbolic groups and CAT(0)-groups and is closed under directed colimits, taking subgroups and direct products. This implies that these conjectures are true for certain interesting groups (Tarsky monsters, groups with expanders, limit groups) and those exotic aspherical manifolds constructed by Mike Davis.

Thursday, February 28
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
LOGICAL PROOF INTERPRETATIONS AS A TOOL FOR “HARD ANALYSIS”

ULRICH KOHLENBACH
Tech. Univ. Darmstadt

Abstract

Historically, proof theory has its origin in Hilbert’s foundational program. Building upon pioneering ideas of G. Kreisel, going back to the 50’s, a new applied form of proof theory emerged during the last 20 year. Here the emphasis is on applications of so-called proof interpretations to concrete mathematical proofs with the aim of extracting effective bounds as well as new uniformity results from prima facie ineffective proofs. This has led to new results in number theory, approximation theory, nonlinear analysis, geodesic geometry and ergodic theory as well as the development of logical metatheorems that explain these results as instances of general logical phenomena. Specialized to the examples discussed in T. Tao’s recent essay “Soft analysis, hard analysis, and the finite convergence principle” the logical machinery yields very much the type of quantitative finitary versions of analytical theorems as considered by Tao. We will argue that these logical methods provide a systematic approach to Tao’s program of “hard analysis”.

Thursday, March 6
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
We say a region of space is “cloaked” with respect to electromagnetic measurements if its contents – and even the existence of the cloak – are inaccessible to such measurements. One recent proposal for achieving cloaking takes advantage of the coordinate-invariance of Maxwell’s equations. I shall explain this scheme, including its mathematical basis and its apparent limitations.
ALGEBRAIC INTEGERS AND ALGEBRAIC DYNAMICS

Curt McMullen
Harvard University

Abstract

We will discuss the role of Hodge theory, Salem numbers and Coxeter groups in the discovery of new dynamical systems on compact complex surfaces.

Thursday, April 10
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
Abstract

There is a rough equivalence between the category of commutative rings and the category of topological spaces, which is the basis of the way in which quantum physics describes the world. Thinking about the equivalence leads us towards variants and generalizations of the objects on both sides of the picture. On the algebraic side we can consider non-commutative rings, but also more subtle kinds of algebraic structures such as quantum field theories. I shall describe how these variants are reflected in the homotopy theory on the geometrical side.
Abstract

The asymmetric simple exclusion process (ASEP) is an interacting particle system that has been much studied both in the mathematics and physics literature. In this lecture we show how using some ideas coming from integrable systems (Bethe Ansatz) various distribution functions for the infinite system can be expressed in terms of certain Fredholm determinants.
Abstract

An example of a free boundary problem is to find the optimal way to insulate an oven or refrigerator. The unknown in this problem is the outer boundary (or so-called free boundary) of the insulating material. We will discuss the regularity theory of energy-minimizing free boundaries: when they are smooth and when they are not. We will then explore the deep analogy with the classical theory of area-minimizing surfaces focusing on the meaning and consequences of stability.
INTERNAL AGGREGATION WITH MULTIPLE SOURCES: 
FROM DIACONIS-FULTON ADDITION TO A FREE 
BOUNDARY PROBLEM

YUVAL PERES
Microsoft Research

Abstract

Start with $n$ particles at each of $k$ points in the $d$-dimensional lattice, and let each particle perform simple random walk until it reaches an unoccupied site. The law of the resulting random set of occupied sites does not depend on the order in which the walks are performed, as shown by Diaconis and Fulton. We prove that if the distances between the starting points are scaled by $n^{1/d}$, then the set of occupied sites has a deterministic scaling limit. In two dimensions, the boundary of the limiting shape is an algebraic curve of degree $2k$. (For $k = 1$ it is a circle, as proved in 1992 by Lawler, Bramson and Griffeath). The limiting shape can also be described in terms of a free-boundary problem for the Laplacian and quadrature identities for harmonic functions. I will show simulations of the process, that suggest several intriguing open problems. Joint work with Lionel Levine.

Thursday, May 8
4:15 p.m.
Room 380-F

http://math.stanford.edu/coll/0708/
Abstract

This will be a survey describing some old questions concerning Riemannian manifolds of positive curvature and their current status. We will give an overview of the methods which have had bearing on them. The main focus will be on positive sectional curvature, but other notions of positivity will also play a role.
Higher structures with connection have been conceived in low dimensionality and/or in sufficiently abelian cases as gerbes with connection or as differential characters. We give a general definition in terms of descent for parallel $n$-volume transport with values in arbitrary structure $n$-groups. Concrete realizations are obtained through $L$-infinity algebra connections and their integration. Examples include Chern-Simons connections and the String-like connections whose existence is obstructed by them.