**Department Colloquium**

**Thursday, December 12**

**4:15pm; Bldg. 380, Room 380-W**

**James Maynard**

*(Université de Montréal)*

“Small gaps between primes”

**ABSTRACT:** It is believed that there should be infinitely many pairs of primes which differ by 2; this is the famous twin prime conjecture. More generally, it is believed that for every positive integer $m$ there should be infinitely many sets of $m$ primes, with each set contained in an interval of size roughly $m \log m$. Although proving these conjectures seems to be beyond our current techniques, recent progress has enabled us to obtain some partial results. We will introduce a refinement of the “GPY sieve method” for studying these problems. This refinement will allow us to show (amongst other things) that $\liminf (p_{n+m} - p_n) < \infty$ for any integer $m$, and so there are infinitely many bounded length intervals containing $m$ primes.
ABSTRACT: The goal of the talk is to survey the emerging field of integrable probability, whose goal is to identify and analyze exactly solvable probabilistic models. The models and results are often easy to describe, yet difficult to find, and they carry essential information about broad universality classes of stochastic processes.
“Fast algorithms for electronic structure analysis”

ABSTRACT: Kohn-Sham density functional theory (KSDFT) is the most widely used electronic structure theory for molecules and condensed matter systems. For a system with $N$ electrons, the standard method for solving KSDFT requires solving $N$ eigenvectors for an $\mathcal{O}(N) \times \mathcal{O}(N)$ Kohn-Sham Hamiltonian matrix. The computational cost for such procedure is expensive and scales as $\mathcal{O}(N^3)$. We have developed pole expansion plus selected inversion (PEXSI) method, in which KSDFT is solved by evaluating the selected elements of the inverse of a series of sparse symmetric matrices, and the overall algorithm scales at most $\mathcal{O}(N^2)$ for all materials including insulators, semiconductors and metals. The PEXSI method can be used with orthogonal or nonorthogonal basis set, and the physical quantities—including electron density, energy, atomic force, density of states, and local density of states—are calculated accurately without using the eigenvalues and eigenvectors. The recently developed massively parallel PEXSI method has been implemented in SIESTA, one of the most popular electronic structure software packages using atomic orbital basis sets. The resulting method can allow accurate treatment of electronic structure in an unprecedented scale. We demonstrate the application of the method for solving graphene-like structures with more than 30,000 atoms, and the method can be efficiently parallelized 10,000-100,000 processors on Department of Energy (DOE) high-performance machines.
ABSTRACT: The two-norms of the resolvents of a normal matrix are completely determined by the spectrum, but a non-trivial amount of computation is required to find the same information from the Schur decomposition of a non-normal matrix. This talk introduces a multi-shift Lanczos variant of S.H. Lui’s inverse iteration scheme which provides two improvements: (1) sequential implementations may achieve near-peak performance due to reduced data movement, and (2) when the Schur factor is too large to fit in a single process’s memory, the large critical path of a triangular solve can be amortized over many right-hand sides.

This new algorithm, coupled with the spectral divide and conquer scheme of Bai, et. al., was recently implemented as part of Elemental, a distributed-memory dense linear algebra library led by the author. Preliminary results for computing the pseudospectra of very large dense matrices will be discussed, along with a brief overview of the author’s recent work on sparse-direct solvers, fast integral transforms, structured matrix arithmetic, and large-scale optimization.
ABSTRACT: How can we understand the structure of an enormous network? For dense graphs, Szemerédi regularity is a fundamental tool that has led to a rich theory of graph limits. However, many real-world networks are sparse, and all sparse graphs converge to zero in the dense theory. To distinguish between them, we need a more refined theory. Bollobás and Riordan took an important step in this direction by analyzing sparse graphs without dense spots, but this hypothesis rules out many important cases such as power-law distributions. In this talk, I’ll review the theory of graph limits and discuss a broad extension to sparse graphs (which is joint work with Christian Borgs, Jennifer Chayes, and Yufei Zhao).
"The arithmetic of hyper elliptic curves"

ABSTRACT: Hyperelliptic curves first appeared in the work of Abel, who studied integrals of their regular differentials. Over the rational numbers, these curves have equations of the form $y^2 = F(x)$, where $F$ is a square-free polynomial with rational coefficients. When the degree of $F(x)$ is at least 5, the genus of the hyperelliptic curve is at least 2 and Faltings has proved that there are only finitely many rational solutions. In this talk, I will discuss some recent results on the average number of rational solutions.
ABSTRACT: This talk is an introduction to “the price of anarchy”: a measure of the inefficiency of selfish behavior that has been successfully analyzed in many applications, including network routing, resource allocation, network formation, auctions, and even models of basketball. It is defined as the worst-case ratio between the welfare of a Nash equilibrium and that of an optimal solution.

The technical results in this talk are two “extension theorems”, which extend near-optimality guarantees for Nash equilibria to more general equilibrium concepts in a “black-box” manner. The first extension theorem applies to easily-learned relaxations of Nash equilibria, and the second to games of complete information.
The Kadison-Singer problem is a question in operator theory which arose in 1959 while trying to make Dirac’s axioms for quantum mechanics mathematically rigorous in the context of von Neumann algebras. It asks whether every pure state on a discrete maximal abelian subalgebra of $B(H)$ extends uniquely to a pure state on all of $B(H)$, where $H$ is a separable complex Hilbert space. In the '70s and '80s, it was realized that the linear-algebraic core of the problem lies in understanding when an arbitrary finite set of vectors in $\mathbb{C}^n$ can be partitioned into two disjoint subsets each of which approximate it spectrally.

We give a positive solution to the problem by proving essentially the strongest possible partitioning theorem of this type. The proof is based on two significant ingredients: a new existence argument, which reduces the problem to bounding the roots of the expected characteristic polynomials of certain random matrices, and a general method for proving upper bounds on the roots of such polynomials. The techniques are elementary, mostly based on tools from the theory of real stable polynomials, and the talk should be accessible to a broad audience.

Joint work with A. Marcus and D. Spielman.

BIO: Nikhil Srivastava is a researcher at Microsoft Research, India. He is currently interested in graphs, linear algebra, probability, optimization, the geometry of polynomials, and various combinations of these things. He received a BS from Union College and a PhD from Yale in 2010 under the guidance of Daniel Spielman. He was a postdoc at the IAS, MSRI, and Princeton before joining Microsoft in 2012.
ABSTRACT: The cyclotomic trace map of Bökstedt-Hsiang-Madsen, is a map from algebraic $K$-theory to a theory called TR. The big de Rham-Witt complex, introduced in joint work with Ib Madsen, bears the same relationship to TR-theory as Milnor $K$-theory bears to Quillen's algebraic $K$-theory. It generalizes the classical $p$-typical de Rham-Witt complex introduced by Bloch-Deligne-Illusie with the purpose of understanding the structure of the Berthelot-Grothendieck crystalline cohomology of regular schemes over $\mathbb{F}_p$. In this talk, I will present a new and explicit construction of the big de Rham-Witt complex, which is based on a theory of modules and derivations over $\Lambda$-rings. A surprising outcome of this new construction is an interpretation of the big de Rham-Witt complex as the complex of differentials along the leaves of a foliation.
Department Colloquium

Thursday, April 24

4:15pm; Bldg. 380, Room 380-W

Richard A. Wentworth
(Maryland)

“The Yang-Mills flow on Kaehler manifolds”

ABSTRACT: The fundamental work of Donaldson and Uhlenbeck-Yau proves the smooth convergence of the Yang-Mills flow of stable integrable unitary connections on hermitian vector bundles over Kaehler manifolds. This was generalized by Bando and Siu to incorporate certain (singular) hermitian structures on reflexive sheaves. Bando-Siu also conjectured what happens when the initial sheaf is unstable; namely, that the limiting behavior should be controlled by the Harder-Narasimhan filtration of the sheaf. In this talk I will describe the resolution of this conjecture, which draws on the work of several authors.
“Blowup or no blowup? The interplay between theory and computation in the study of 3D Euler equations”

**Abstract:** Whether the 3D incompressible Euler equations can develop a singularity in finite time from smooth initial data is one of the most challenging problems in mathematical fluid dynamics. This is closely related to the Clay Millennium Problem on 3D Navier-Stokes Equations. We first review some recent theoretical and computational studies of the 3D Euler equations. Our study suggests that the convection term could have a nonlinear stabilizing effect for certain flow geometry. We then present strong numerical evidence that the 3D Euler equations develop finite time singularities. A careful local analysis also suggests that the blowing-up solution is highly anisotropic and is not of Leray type. A 1D model is proposed to study the mechanism of the finite time singularity. Very recently we prove rigorously that the 1D model develops finite time singularity.
"Energy, momentum, and center of mass in general relativity"

**Abstract:** These notions in the title are of most fundamental importance in any branch of physics. However, there have been great difficulties in finding physically acceptable definitions of these concepts in general relativity since Einstein’s time. I will explain these difficulties and progresses that have been made. In particular, I will introduce new definitions of center of mass and angular momentum at both the quasi-local and total level, which are derived from first principles in general relativity and by the method of geometric analysis. With these new definitions, the classical formula $p = mv$ is shown to be consistent with Einstein’s field equation for the first time. The talk is based on joint work with Po-Ning Chen and Shing-Tung Yau.
Department Colloquium

Thursday, May 29

4:15pm; Bldg. 380, Room 380-W

Gautam Iyer
(Carnegie Mellon)

“Stirring and Mixing”

ABSTRACT: I will talk about various “mixing” questions that have attracted interest recently. For instance, “How well can you stir cream into your coffee, and at what cost?”. Or more pathologically “Can you stir your coffee to keep it hot for longer?”. One aspect of this I will focus on mathematically translates into studying the decay of negative Sobolev norms of the transport equation. This also involves very interesting connection to Bressan’s (still open!) rearrangement cost conjecture. I will spend most of the talk surveying recent results, and conclude with brief description of joint work with A. Kiselev, Xiaoqian Xu and myself.
ABSTRACT: What is the canonical way to choose a random, discrete, two-dimensional manifold which is homeomorphic to the sphere? One procedure for doing so is to choose uniformly among the set of surfaces which can be generated by gluing together $n$ Euclidean squares along their boundary segments. This is an example of what is called a random planar map and is a model of what is known as pure discrete quantum gravity. The asymptotic behavior of these discrete, random surfaces has been the focus of a large body of literature in both probability and combinatorics. This has culminated with the recent works of Le Gall and Miermont which prove that the $n \to \infty$ distributional limit of these surfaces exists with respect to the Gromov-Hausdorff metric after appropriate rescaling. The limiting random metric space is called the Brownian map.

Another canonical way to choose a random, two-dimensional manifold is what is known as Liouville quantum gravity (LQG). This is a theory of continuum quantum gravity introduced by Polyakov to model the time-space trajectory of a string. Its metric when parameterized by isothermal coordinates is formally described by $e^{\gamma h}(dx^2 + dy^2)$ where $h$ is an instance of the continuum Gaussian free field. LQG for $\gamma = \sqrt{8}/3$ is conjecturally equivalent to the Brownian map and to the limits of other discrete theories of quantum gravity for other values of $\gamma$.

In this talk, I will first survey this literature and then describe a new family of growth processes called quantum Loewner evolution (QLE) which we propose using to endow LQG with a distance function which is isometric to the Brownian map. I will also explain how QLE is related to diffusion limited aggregation, the dielectric breakdown model, and the Schramm-Loewner evolution. The second part of this talk is based on joint works with Scott Sheffield.