General Relativity & Shock Wave Theory

A conference in honor of Joel Smoller on his 70th Birthday

Stanford University
April 29 – May 1, 2006

CONFERENCE LOCATION
Sloan Mathematics Center, Building 380
Saturday & Sunday (4/29 & 4/30)
Mitchell Building Monday (5/1)

PLENARY SPEAKERS
Alberto Bressan (Penn State University)
Felix Finster (University of Regensburg, Germany)
James Glimm (Stony Brook University)
Peter Lax (Courant Institute/New York University)
Richard Schoen (Stanford University)
Shing-Tung Yau (Harvard University)
Joel Smoller (University of Michigan)

ORGANIZING COMMITTEE
Robert Gardner (University of Massachusetts)
David Hoff (Indiana University)
Tai-Ping Liu (Stanford University)
Blake Temple (UC-Davis)
Zhouping Xin (Chinese University of Hong Kong)

CONTACT
Conference Coordination and Web site—
Pat Cahill (cahill "at" math.stanford.edu)
Invited Speakers & Participants

- Alberto Bressan (Penn State University)
- Gui-Qiang Chen (Northwestern University)
- Joe Conlon (University of Michigan)
- Volker Elling (Brown University)
- Paul Federbush (University of Michigan)
- Razvan Fetecau (Stanford University)
- Felix Finster (University of Regensburg, Germany)
- Xabier Garaizar (Lawrence Livermore National Laboratory)
- Robert Gardner (University of Massachusetts)
- James Glimm (State University of New York, Stony Brook)
- James Greenberg (Carnegie Mellon University)
- Seungyeal Ha (Seoul National University, South Korea)
- David Hoff (Indiana University)
- John Hunter (University of California, Davis)
- Thomas Hou (California Institute of Technology)
- Kris Jenssen (Penn State University)
- Shi Jin (University of Wisconsin)
- Niky Kamran (McGill University, Canada)
- Barbara Lee Keyfitz (University of Houston)
- Peter Lax (Courant Institute/New York University)
- Ho Lee (Seoul National University, South Korea)
- Ming-Yi Lee (Stanford University)
- Doron Levy (Stanford University)
- David Li (Citigroup)
- Tianhong Li (Stanford University)
- Tong Li (University of Iowa)
- Tai-Ping Liu (Stanford University)
- Tao Luo (Georgetown University)
- Gregory Lyng (University of Wyoming)
- Shinya Nishibata (Tokyo Institute of Technology, Japan)
- Ronghua Pan (Georgia Institute of Technology)
- Richard Schoen (Stanford University)
- Marshall Siemrod (University of Wisconsin)
- Joel Smoller (University of Michigan)
- Alexander Sotirov (Stanford University)
- Eitan Tadmor (CSCAMM, University of Maryland)
- Blake Temple (University of California, Davis)
- Eugene Tsyganov (North Carolina State University)
- Zeke K. Vogel (University of California, Davis)
- David Wagner (University of Houston)
- Brian Wissman (University of California, Davis)
- Tong Yang (City University of Hong Kong)
- Shing-Tung Yau (Harvard University)
- Robin Young (University of Massachusetts)
- Shih-Hsien Yu (City University of Hong Kong)
- Seok Bae Yun (Seoul National University, South Korea)
- Yanni Zeng (University of Alabama)
- Yong Zhou (East China Normal University)
- Kevin Zumbrun (Indiana University)

Featured speakers indicated in blue.
<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00–9:10</td>
<td>Opening remarks</td>
</tr>
<tr>
<td>9:10–10:00</td>
<td><strong>Peter Lax</strong>—Courant Institute/New York University</td>
</tr>
<tr>
<td></td>
<td><strong>Title:</strong> A Phragmen-Lindelof and Saint Venant Principle in Harmonic Analysis</td>
</tr>
<tr>
<td></td>
<td><strong>Abstract:</strong> Let $S$ be a linear space of vector valued functions $u(y)$ on the half-line whose values belong to some Banach space. We suppose that $S$ is translation invariant; that is, if $u(y)$ belongs to $S$, so does $u(y+t)$ for all $t&gt;0$. $S$ is called &quot;interior compact&quot; if the unit ball of $S$ in the $L_1$ norm over a $y$-interval $[a,b]$ is precompact in the $L_1$ norm over any proper subinterval $[a',b']$.</td>
</tr>
<tr>
<td></td>
<td><strong>Theorem:</strong> Any function $u(y)$ in a translation invariant, interior compact space that is $L_1$ on $y&gt;0$ decays exponentially as $y$ tends to infinity, and has an asymptotic expansion near infinity in terms of exponential functions in $y$ contained in $S$.</td>
</tr>
<tr>
<td></td>
<td><em>This result can be applied to solutions of elliptic equations in a half cylinder.</em></td>
</tr>
<tr>
<td></td>
<td><strong>Chair:</strong> Tai-Ping Liu</td>
</tr>
<tr>
<td>10:00–10:30</td>
<td>Break</td>
</tr>
<tr>
<td>10:30–12:30</td>
<td><strong>Robert Gardner</strong>—University of Massachusetts</td>
</tr>
<tr>
<td></td>
<td><strong>Title:</strong> A Numerical Method for the Selection of Deterministic Reaction-Diffusion Models of Noisy Experimental Data in Population Biology</td>
</tr>
<tr>
<td></td>
<td><strong>Abstract:</strong> Predator-herbivore-host plant interactions that frequently arise in agriculture when the predators and their prey are specialists occur as complex successions of localized patches of rapid mutual growth and decline. Specialists have evolved in a manner that enables each species to exploit the dispersive behavior driven by their search for new food sources following a localized crash.</td>
</tr>
<tr>
<td></td>
<td>Experimental studies of sustained, large-scale interactions are often extremely noisy, and there are often many time and length scales that can be discerned in connection with local and subregional patterns of growth and decline. Biologists therefore often rely on stochastic systems as mathematical models of the resulting population dynamics.</td>
</tr>
<tr>
<td></td>
<td>Nevertheless, there are often larger scale, coherent patterns and waves that can be observed to occur both in experimental data and stochastic mathematical models. In this talk, we describe a class of deterministic reaction-diffusion models that may be useful in understanding the time and length scales associated to certain coherent, wave-like behavior in such locally &quot;self-annihilating&quot; systems. We also present a numerical method for</td>
</tr>
</tbody>
</table>
determining the rate functions and diffusion coefficients in the system that involves the minimization of certain functionals calculated from experimental spatio-temporal data sets.

**Tong Li**—University of Iowa  
**Title:** Transition to Instabilities of Discrete Shocks  
**Abstract:** We study transition to instabilities of discrete shocks. It is found that the discrete dynamic system describing the discrete shock waves exhibits rich dynamics resulting from the combined effects of discretization, nonlinearity, and dissipation. The onset of the instability of discrete shocks is identified and the stability threshold is in terms of shock wave strength, mesh size, and viscosity.

**Tong Yang**—City University of Hong Kong  
**Title:** The Boltzmann Equation in the Space $L^2 \cap L^\infty$: Global and Time-Periodic Solutions  
**Abstract:** We present a function space in which the Cauchy problem for the Boltzmann equation is well-posed globally in time near an absolute Maxwellian in a mild sense without any regularity conditions. The asymptotic stability of the absolute Maxwellian is also established in this space and, moreover, it is shown that the higher order spatial derivatives of the solutions vanish in time faster than the lower order derivatives. No smallness assumptions are imposed on the derivatives of the initial data, and the optimal decay rates are derived. Furthermore, the Boltzmann equation with a time-periodic source term is solved in the same space on the unique existence and stability of a time-periodic solution which has the same period as the source term. The proof is based on the spectral analysis of the linearized Boltzmann operator. This is a joint work with Seiji Ukai.

**Volker Elling**—Brown University  
**Title:** Exact Solutions for Supersonic Flow onto a Solid Wedge  
**Abstract:** Supersonic flow onto a solid wedge has steady solutions with a straight shock emanating downstream from the wedge tip. But the corresponding shock relations have two different solutions, a weak and a strong shock. We show some numerical simulations discussing which shock is stable in which sense. In addition, we study a test problem where the upstream state fills the entire domain at $t=0$. We construct an exact unsteady (selfsimilar) solution, for potential flow as model, in which the weak shock appears spontaneously at the tip.

**Chair:** Tom Hou
<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:30–2:00</td>
<td>Lunch break</td>
</tr>
</tbody>
</table>
| 2:00–4:00 | Xabier Garaizar—Lawrence Livermore National Laboratory  
**Title:** Applied Mathematics in CASC  
**Abstract:** The Center for Applied Scientific Computing (CASC) conducts collaborative scientific investigations that require the power of high performance computers and the efficiency of modern computational methods. This talk will focus on several examples of innovative use of applied mathematics methods to solve problems of interest to some of our sponsors, most notably in the Department of Energy.  
Shih-Hsien Yu—City University of Hong Kong  
**Title:** Green’s function for Boltzmann equation and its Applications  
**Abstract:** In this talk we will survey the recent mathematical theories for Boltzmann equation, in particular, the Green’s functions for Boltzmann equation. There are several highlights in this talk: macro-micro decomposition, positive solution of the Boltzmann shock layer, hydrodynamic limit problem, Boltzmann boundary layer, particle-wave decomposition, mixture lemma, and the Green’s functions for both initial value problem and initial-boundary value problems.  
Seungyeal Ha—Seoul National University  
**Title:** Remarks on the Stability of the Boltzmann Equation  
**Abstract:** In this talk, I will discuss the $L^1$-stability of the Boltzmann equation. The analysis depends on the explicit construction of a functional which is equivalent to $L^1$-distance, and Green’s function approach with detailed pointwise behavior of solutions.  
Eugene Tsyganov—North Carolina State University  
**Title:** Global existence and asymptotic convergence of weak solutions for the one-dimensional Navier-Stokes equations with capillarity and nonmonotonic pressure  
**Abstract:** We construct global weak solution of the Navier-Stokes equations with capillarity and nonmonotonic pressure. We show that the velocity variable $u \to 0$ strongly in $L^2 ([0,1])$ and the volume variable $v$ approaches the set of stationary solutions in $H^1 ([0,1])$ asymptotically in time.  
**Chair:** Marshall Slemrod |
| 4:00–4:30 | Break                                   |
4:30–5:20  ● Shing-Tung Yau—Harvard University
   **Title:** Spacetime with Torsion
   **Abstract:** In this talk, I will discuss the work that I did with Fu on constructing a supersymmetric spacetime with flux. One needs to solve a complex Monge ampere-type equation on a non-Kahler manifold to achieve such a model. It is currently being pursued for its physical meaning.
   **Chair:** Leon Simon

6:30  RECEPTION & BANQUET at the Westin Hotel

---

**Sunday, April 30**

**Sloan Mathematics Building, Room 380-W**

9:00–9:50  ● James Glimm—State University of New York, Stony Brook
   **Title:** Front Tracking and Its Applications
   **Abstract:** Recent improvements to the Front Tracking method have increased considerably its power and its ease of use. Some of the more important of these developments are outlined here.

   The problem of robust treatment of intersections and bifurcations has been resolved: we cut out a small region around the bifurcation and there we use a robust but less accurate method (grid based tracking).

   The combined algorithm is called Locally Grid Based Tracking. Improved accuracy also results from a fully conservative tracking algorithm, replacing the ghost fluid algorithm invented by the author and coworkers in 1980.

   Use of these improved algorithms and other upgrades allows accurate simulation of several challenging problems from physics. Especially noteworthy is the classical problem of validation of simulations for the 3D Rayleigh-Taylor chaotic mixing problem. We improved the physics modeling in addition to the numerical modeling mentioned above, to get excellent agreement with experiment in the overall growth rate, in contrast to the usual factor of two discrepancy.

   It is a pleasure to thank the many collaborators and colleagues who contributed to the work to be presented.
   **Chair:** Gui-Qiang Chen

9:50–10:20  Break

10:20–12:20  ● Kevin Zumbrun—Indiana University
   **Title:** Relative Hopf bifurcation and galloping instabilities in detonation fronts
   **Abstract:** We discuss, from several points of view, the transition to
instability of viscous detonation waves in one space dimension. The
generic picture that emerges is that of a relative Hopf bifurcation with
respect to the group invariance of translation, corresponding with
“galloping” or “pulsating” instabilities observed in experiment.

- **Yanni Zeng**—University of Alabama
  **Title:** Gas Flow with Several Nonequilibrium Modes
  **Abstract:** We generalize my previous work on gas flow with one
  nonequilibrium mode to a more realistic model that includes several
  nonequilibrium modes. Similar results on fundamental solutions, global
  existence and large time behavior for small solutions can be obtained
  similarly.

- **Shinya Nishibata**—Tokyo Institute of Technology
  **Title:** Asymptotic Behavior of Solutions to the Quantum Hydrodynamic
  Model for Semiconductors
  **Abstract:** Recently a quantum effect becomes more important for the
  analysis on the behavior of electron flows in semiconductor devices as they
  become truly minute. In this talk, we discuss the hydrodynamic model for
  semiconductors with the quantum effect over a one-dimensional bounded
  domain and show the asymptotic stability of an stationary solution to this
  model. Precisely, we prove that the stationary solution uniquely exists and
  then the solution to the non-stationary problem converges to the
  corresponding stationary solution exponentially fast as time tends to infinity
  provided that the difference of the initial data and the stationary solution is
  small in the suitable Sobolev space.

  These results are obtained through the joint research with Mr. Masahiro
  Suzuki at Tokyo Institute of Technology.

- **Alexander Sotiropov**—Stanford University
  **Title:** Boundary Convergence of Hard Spheres with Zero Temperature
  Background
  **Abstract:** We study the fundamental problem of two gas species whose
  molecules collide as hard spheres in the presence of a flat boundary and
  with dependence on only one space dimension. More specifically the linear
  problem arising when the second gas dominates as a flow with constant
  velocity (and hence zero temperature) is considered. The boundary
  condition adopted consists of prescribing the outgoing velocity distribution
  at the wall. It is discovered that the presence of the boundary under
  general assumptions on the outgoing distribution ensures the convergence
  of a series of path integrals and thus a convenient representation for the
  solution is obtained.

  **Chair:** Eitan Tadmor

<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:20–2:00</td>
<td>Lunch break</td>
</tr>
<tr>
<td>Time</td>
<td>Speaker</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------------------</td>
</tr>
<tr>
<td>2:00–4:00</td>
<td>David Wagner</td>
</tr>
<tr>
<td>2:00–4:00</td>
<td>Robin Young</td>
</tr>
<tr>
<td>2:00–4:00</td>
<td>Yong Zhou</td>
</tr>
<tr>
<td>2:00–4:00</td>
<td>David Li</td>
</tr>
<tr>
<td>2:00–4:00</td>
<td></td>
</tr>
<tr>
<td>Break</td>
<td></td>
</tr>
<tr>
<td>4:30–5:20</td>
<td>Alberto Bressan</td>
</tr>
</tbody>
</table>
However, under a monotonicity assumption on the wave speeds, the ODE has bounded variation in the direction of a suitable cone, and unique global solutions are obtained.

**Chair:** Shi Jin

<table>
<thead>
<tr>
<th>Monday, May 1</th>
<th>Mitchell Earth Sciences Building, Room B67</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9:00–9:50</strong></td>
<td><strong>Richard Schoen</strong>—Stanford University</td>
</tr>
<tr>
<td><strong>Title:</strong> Angular Momentum in General Relativity</td>
<td></td>
</tr>
<tr>
<td><strong>Abstract:</strong> In this lecture we will describe asymptotic conditions for solutions of the vacuum Einstein constraint equations which form dense subsets of Sobolev spaces of general asymptotically flat solutions. We use these to prove a deformation theorem for the angular momentum; specifically we describe a procedure to input large amounts of angular momentum into vacuum solutions while keeping the energy and linear momentum roughly constant.</td>
<td></td>
</tr>
<tr>
<td><strong>Chair:</strong> James Greenberg</td>
<td></td>
</tr>
<tr>
<td><strong>9:50–10:10</strong></td>
<td>Break</td>
</tr>
<tr>
<td><strong>10:10–11:00</strong></td>
<td><strong>Felix Finster</strong>—University of Regensburg</td>
</tr>
<tr>
<td><strong>Title:</strong> A Variational Principle in Discrete Space-Time</td>
<td></td>
</tr>
<tr>
<td><strong>Abstract:</strong> The &quot;principle of the fermionic projector&quot; provides a new model of space-time together with the mathematical framework for the formulation of physical theories. It was proposed to formulate physics in this framework based on a particular variational principle. The talk begins with a brief physical motivation, then the mathematical framework is introduced from the basics. Our variational principle is set up and explained in simple examples. Recent results and open problems are discussed.</td>
<td></td>
</tr>
<tr>
<td><strong>Chair:</strong> Blake Temple</td>
<td></td>
</tr>
<tr>
<td><strong>11:00–11:20</strong></td>
<td>Break</td>
</tr>
<tr>
<td><strong>11:20–12:20</strong></td>
<td><strong>David Hoff</strong>—Indiana University</td>
</tr>
<tr>
<td><strong>Title:</strong> Lagrangean Structure and Propagation of Singularities in Multidimensional Compressible Flow</td>
<td></td>
</tr>
</tbody>
</table>
| **Abstract:** We study the Lagrangean structure of solutions of the Navier-Stokes equations of multidimensional compressible flow in an intermediate regularity class in which initial energies are small and densities are essentially bounded. We prove that there is a unique particle trajectory emanating from each point of any open set in physical space in which the initial density is strictly positive and that such open sets are convected
homeomorphically by the flow. As corollaries we show that Holder continuous surfaces are transported into Holder continuous surfaces, that sectional continuity of the density and the divergence of the velocity are preserved, that the Rankine-Hugoniot conditions hold in a strict, pointwise sense across such surfaces, and that the strengths of singularities decay exponentially in time when the pressure is a monotone function of density. These results require that initial velocities are in certain fractional Sobolev spaces depending on the dimension, and we display an example indicating that this requirement is necessary.

Blake Temple—University of California, Davis

TITLE: How a Finite Mass Cosmology Might Emerge from an Inflationary Spacetime

ABSTRACT: We consider how the finite mass shock wave cosmology introduced by the authors in PNAS, Vol. 100, could connect up with Guth's original theory of inflation.

In our original shock wave model, a spacetime of constant density and constant pressure emerges instantaneously after the Big Bang, just as in the standard model. But in the shock wave model, we assume that the total mass is finite at each fixed time by incorporating a finite mass cut-off far out ... even so, in the refined model, the density and pressure are everywhere constant instantaneously after the Big Bang, just as in the standard model. As a consequence of the finite mass cutoff, the density and pressure drop faster far out than in the Friedmann universe which is close in, and this produces a gradient in the density and pressure, which then breaks into a wave.

In the shock wave model, we describe the evolution of the outer pressure so that this wave is exactly resolved by a single, outgoing, spherical, entropy-satisfying shock wave. Thus, the shock wave emerges from the center of the explosion at the instant of the Big Bang as a zero strength wave that strengthens as it propagates outward, (something like the blast wave of a nuclear explosion), and the expanding galaxies correspond to the region inside the wave. In this talk we discuss how such a finite mass cosmology might arise in a natural way from the standard inflationary universe.

CHAIR: Barbara Lee Keyfitz

12:20–2:00 Lunch break

2:00–2:50 Joel Smoller—University of Michigan

TITLE: With a Little Help From My Friends

CHAIR: Tai-Ping Liu

Joel’s presentation—watch it here
### A few notes

#### Access to Computers:

Our public computer stations are available for your use while you are here. The computer room is located on the third floor of the Math building (Building 380), room 383-P, facing the elevator. The passcode is 70351.

To use any computer there:
- user = public
- password = math383p

#### Places to eat lunch:

Over the weekend, not surprisingly, very few places to eat are open on campus. Two that we know about are:
- Subway sandwiches, located at Tresidder Memorial Union
- The Cool Café, located at Cantor Arts Center

Of course, on Monday, May 1st, classes will be in session and you can take your pick from the many dining options available to the Stanford community.

As for where to eat breakfast and dinner, we encourage you to explore downtown Palo Alto/University Avenue and beyond, to find the meal that will truly “hit the spot.”

#### Parking:

Parking will be enforced on Monday, from 8:00 a.m. to 4:00 p.m. Visitors have several payment options for parking:

- Visitor parking is available in “The Oval,” at the rate of $1.50/hour (in quarters)
- One-day "Visitor" scratcher permits (which allow parking in Pay & Display lots and at meters), available for $12 each at the Parking & Transportation office (open weekdays 7:30 a.m. to 5 p.m.)
Parking permit machine in front of the Cantor Art Center

You may purchase up to 8 hours of parking. The machines accept credit cards, cash or coins. You should have the correct amount of money, since these machines do not issue change. The receipt should be placed on the car dashboard.

The Math department has a limited number of parking passes available. Please ask our receptionist, Rose Stauder, for assistance.

The Ruth Wattis Mitchell Earth Sciences Building

NOTE: Our Monday session takes place not in the Math building, but in the Mitchell Earth Sciences building. As you can see from the map, above right, the Mitchell building is at the top of the hill on Lomita Mall, up from the Math “Corner” and is easy to find.

Once you’ve arrived at the Mitchell building, cross the plaza and enter the lobby. Take the elevator (on your left) and go to the basement.

When you exit the elevator, go to your left. Turn right at the next hallway, and proceed to the very end (past the vending and Coke machines). The door to room B67 is at the end of the hall.

Photos from the conference Watch the slide show