The Mathematics Research Center
Distinguished Lecturer Series
presents

Laure St Raymond (École Normale Supérieure)
From particle systems to kinetic equations

LECTURE 1: Wednesday, February 25–4:15PM in 380-C
“The low density limit: formal derivation”
Consider a deterministic system of $N$ hard spheres of diameter $\varepsilon$. Assume that they are initially independent and identically distributed. Then, in the limit when $N \to \infty$ and $\varepsilon \to 0$ with $N\varepsilon^2=1$ (Boltzmann-Grad scaling), the one-particle density can be approximated by the solution to the kinetic Boltzmann equation. In particular, particles remain asymptotically independent. In this first lecture, we will present the formal derivation of this low density limit, and discuss two important features, namely the propagation of chaos and the appearance of irreversibility.

LECTURE 2: Thursday, February 26–4:15PM in 380-W
“A short time convergence result”
Lanford’s theorem states that in the Boltzmann-Grad limit the one-particle density converges to the solution of the kinetic Boltzmann equation almost everywhere on a short-time interval (corresponding actually to a fraction of the average first collision time). The proof relies on a careful study of the recollision mechanism (which is not described by the Boltzmann dynamics), and on a priori bounds obtained by a Cauchy-Kowalewski argument.

In this second lecture, we will give a sketch of this proof, and show that the time restriction is due to the lack of global a priori bounds.

(Reception at 3:00pm, before the talk, in the new 4th floor lounge!)

LECTURE 3: Friday, February 27–10:30AM in 383-N
“The case of fluctuations around a global equilibrium”
To improve the convergence time, the idea is therefore to consider situations where one can obtain global a priori bounds. Since the system of particles is Hamiltonian, the Gibbs measures are invariant under the flow and provide constant solutions to the Boltzmann equation. By perturbation, we get initial data leading to linear kinetic equations. In this third lecture, we will show that, for such initial data, we can control a priori the collision rate, and extend Lanford’s strategy to get a quasi-global convergence result in the low density limit.