Stanford Undergraduate Research Institute in Mathematics

June 25-August 17, 2012

The Stanford Undergraduate Research Institute in Mathematics is an eight week program that provides Stanford undergraduates the opportunity to work on mathematical problems in an extra-curricular context. Most students will work on interesting mathematical problems in a collaborative environment. A number will work one-on-one with faculty member. Summer funding will be available for some students, thanks to VPUE; others can obtain course credit in the fall quarter for participating. Those who obtain summer funding will receive a stipend of $4800 for the eight weeks.

Individual Research with a Faculty Member

Students working individually with a faculty member will decide on a project and the dates in consultation with their faculty adviser. Note that it is the duty of the student to find a faculty member interested and willing to work with them. A short project proposal will be requested with the application.

Collaborative Research

The remaining students will take part, full time, in the eight-week program that will run from Monday, June 25 through Friday, August 17.

Goal of the program

At SURIM students will be exposed to questions that are of interest in current mathematics, as well as the research and exploration aspects that accompany such questions. With their mentor's assistance, students will study the prerequisite materials to understand their program's topic and will then participate in exploration of their questions about the subject. The emphasis will be on self discovery of examples and properties. In addition to knowledge of their subject and an understanding of what it means to explore a research question, participants will
practice the ability to present mathematics in a formal seminar setting, use software such as LaTeX to typeset mathematics, use other programming languages to study mathematical questions, and interact with peers, graduate students and faculty.

Eligibility

All Stanford students who will be enrolled full/part time during the Fall of 2012 are eligible to apply.

Format for the eight-week program

Students will be divided into groups depending on their mathematical interest and background. Each group will work closely with graduate students.

A typical week

There will be a couple of formal meetings with mentors each week. At the start, the mentors will lay out the beginning of the project, and the groups will decide how best to begin. Each group will prepare presentations to the entire institute each week, giving a status report to those working on other problems. (Practice with getting across ideas is essential to doing mathematics!)

Much of the week will be spent working individually and in groups, and in informal discussions with mentors.

There will be roughly two additional events per week. Some will be introductions to research tools (from writing with latex to the use of various software packages). Others will be lectures from researchers in academia and industry on what research is actually about — how it is done, how to do it, and what it is like. The exact details can be found in the calendar of events. The SURIM group will also have access to various classrooms during the summer, see the space/time schedule.

Projects

We will likely divide into three groups.

Can you hear the shape of a drum?

In this project we will investigate what happens to a shape in the plane when it is
"drummed." This exploration leads to the natural question of whether one can recover the shape of the drum from the sounds it makes. We'll learn how this is can be viewed as a question about the eigenvalues of the Laplacian, one of the simplest partial differential operators. In particular we will attempt to construct isospectral non-isometric polygons (planar polygons which do not look the same but support the same noises when drummed). Students will learn about constructions of such examples and explore the wealth of existing and new examples to examine what properties are "spectrally determined," using combinatorial, analytic and numerical (computer) methods. Other possible research directions include studying isospectral flat tori, which leads to an exploration of lattices and their length spectrum. This program has limited prerequisites, a student who has background in multivariable calculus and linear algebra, at the level of the Math 50, series will find the material accessible. Students with more focused interests in analysis, PDEs, or geometry will find that the subject material lends itself easily to more advanced questions as well. Participants should expect to learn about infinite dimensional eigenvalue problems, combinatorial, analytic, and numerical methods in spectral geometry, lattices and much more! More information can be found here!

**Optimal transportation (guided by Prof. Yanir Rubinstein)**

Suppose you are given a detailed map of California's lakes and ponds, and another map with the state's water consumption per town. What would be the most efficient water canal system that would transport the available water to all inhabitants? The theory of Optimal Transportation grew out of questions of this sort, whose earliest treatment is probably that of the French scientist Gaspard Monge in 1781, employed by the French army at that time. Since then much progress has been made and the mathematical theory has greatly evolved. Nowadays, optimal transportation techniques have found many deep and unexpected applications ranging from differential geometry (including, e.g., Ricci flow), probability, partial differential equations, economics, and even meteorology. At the same time, there are fascinating open problems, some of which are rather geometric, and are not hard to formulate and understand with little background. Students working in this project will be expected to learn the basics of the theory in the first few weeks, through guided reading and a working seminar where students will lecture to each other. In the second part of the program the students will be working on research problems. Some of these problems will also involve an experimental component where computer programming/visualization could be useful. This project will be overseen by Dr. Yanir Rubinstein who will suggest the problems (based to some extent by the students interests and background) and give some presentations during the summer. The students will be mentored on a regular basis by a graduate student (Otis
Chodosh). Prerequisites: Students should be familiar with the basics of measure theory and some functional analysis (although the latter is not absolutely necessary). Familiarity with the basics of partial differential equations would be advantageous, but is not necessary. Computer programming skills (Matlab, Maple or Mathematica) could prove useful as well.

**Explorations in Number Theory**

In this project students will focus on two areas of number theory. Depending on the students interests, this could include studying the class groups of quadratic number fields (which measure the failure of unique prime factorization and are related to binary quadratic forms) and examining the remarkable number-theoretic properties of integer linear recurrences. Part of the class group project will be computational, students will learn to compute both by hand and via the use of computer algebra system SAGE. Students will notice some patterns, and try to prove that they hold in general. For example, there is a relationship between 2-torsion in the class group and the factorization of the discriminant. The integer linear recurrence project will involve examining the rich literature on zeros of linear recurrence sequences, including the beautiful theorem of Mahler-Skolem-Lech. Students will analyze the periodicity and divisibility properties of linear recurrence sequences, and relations to linear algebra, Galois theory, computability theory, and combinatorics. For example, students can explore the question of what is a sharp bound on the periodicity of linear recurrences mod n, and when is it attained? Prerequisites are minimal; both projects have aspects ranging from the immediately accessible to the bleeding edge of mathematics. A background in linear algebra, algebraic number theory, Galois theory, or complex analysis would open up more avenues for exploration, but are not required.

**Application**

Please submit the following information, by email, to Nancy Rodriguez at nrodriguez@math.stanford.edu, by March 1, 2012 with subject line "SURIM application". Please include in the email the following information:

- (a) Name and year.
- (b) If you have a faculty member who has agreed to work one-on-one with you, please let us know. (This is not necessary to apply.) If this is the case, please include a short proposal, developed in consultation with your intended mentor.
- (c) Name of one or two professors who are familiar with you (ideally in
mathematics)
- (d) Mathematical background and interests.
- (e) For those not working individually with a faculty member, which of the possible projects appeal to you?
- (f) Do you need funding in order to take part? Would you like course credit? (Note: it is not possible to get both funding and credit.)
- (g) Curriculum vitae and unofficial Stanford transcript.
- (h) Project proposal for those seeking to work one-on-one with a faculty member.

Notification Deadline: Students will be notified of their acceptance by March 15, 2012.

People

Directors: Gunnar Carlsson and Nancy Rodriguez, Assistant Director: Ravi Vakil.

Mentors: Jeremy Booher, Otis Chodosh, Chris Henderson, Seungki Kim, Sam Lichtenstein, Daniel Litt, Cary Malkiewich, Khoa Nguyen, Simon Rubinstein-Salzedo.

Questions?

If you have any questions, or are even just curious about the program, please contact Dr. Nancy Rodriguez (nrodriguez@math.stanford.edu). She will also be available to chat during iDeclare week.