Since the early days of ergodic theory and the pioneering work von Neumann, ergodic theory and harmonic analysis have been intimately connected, often in surprising ways. A classic example of this is the use of harmonic analysis to prove convergence results for ergodic averages. Recently, there has been a great deal of interplay between these two fields, including work motivated by applications to Ramsey Theory. This interplay also touches upon subjects of active interest in probability theory, such as martingales, Bernoulli convolutions and random polynomials, as well as the Kakeya problem which is one of the outstanding open problems in harmonic analysis. This conference will bring together many of the leading mathematicians in these related areas to report on recent developments of broad interest and to point the way for exciting directions for future research. In this way we plan to honor the significant contributions of Izzy Katznelson in these areas on the occasion of his 70th Birthday.
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OPEN PROBLEMS (gathered by N. Berger, A. Bufetov, C. Demeter and N. Frantzikinakis)

I. Assani. Let $(X, B, \mu)$ be a finite measure space, $T_i$ be measure preserving transformations acting on $X$, and $f_i$ be bounded functions, $i = 1, \ldots, 2^k - 1$. The averages along the cubes for three terms ($k = 2$) and seven terms ($k = 3$) are defined as follows:

$$M_N(f_1, f_2, f_3)(x) = \frac{1}{N^2} \sum_{m,n=1}^N f_1(T_1^m x) \cdot f_2(T_2^n x) \cdot f_3(T_3^{m+n} x)$$

and

$$M_N(f_1, f_2, \ldots, f_7)(x) = \frac{1}{N^8} \sum_{m,n,p=1}^N f_1(T_1^m x) \cdot f_2(T_2^n x) \cdot f_3(T_3^p x) \cdot f_4(T_4^{m+n} x) \cdot f_5(T_5^{m+p} x) \cdot f_6(T_6^{n+p} x) \cdot f_7(T_7^{m+n+p} x).$$

Similarly, we define the averages along the cubes for $2^k - 1$ terms and we denote them by $M_N(f_1, f_2, \ldots, f_{2^k-1})$. The averages along cubes for a single transformation (i.e. $T_i = T$ for all $i$) were introduced by V. Bergelson who proved $L^2$ convergence for $k = 2$. His result was extended by B. Host and B. Kra who proved $L^2$ convergence for $k > 2$ (for a single transformation). I. Assani proved a.e. convergence in this case.

**Problem 1.** Do the averages $M_N(f_1, f_2, \ldots, f_{2^k-1})$ converge in $L^2$ and a.e. for every $k \in \mathbb{N}$?

Assuming that the transformations $T_i$ are ergodic and commute, I. Assani remarks that the answer is yes for every $k$, and without further assumptions the answer is yes for $k = 2$. If the transformations are weak mixing (not necessarily commuting) then the limit (a.e. and in $L^2$) exist and equals the product of the integrals.

**Problem 2.** Find characteristic factors for $L^2$ and a.e. convergence for the averages $M_N(f_1, f_2, \ldots, f_{2^k-1})$.

J. Campbell. For a sequence $\{T_n\}$ of operators and $n_1 < n_2 < \ldots < n_k < \ldots$ define the oscillation operator

$$O_2(T_n, f, \{n_k\})(x) := \left( \sum_{k=1}^\infty \sup_{n_k < n \leq n_{k+1}} |T_{n_k} f(x) - T_n f(x)|^2 \right)^{\frac{1}{2}},$$

and the $p$-variation operator

$$V_p(T_n, f)(x) = \sup_{\{n_k\}} \left( \sum_k |T_{n_k} f(x) - T_{n_{k+1}} f(x)|^p \right)^{\frac{1}{p}},$$

$p > 2$.

An easy exercise shows that if $O_2(T_n, f)(x) < \infty$ a.e. $x$ for each $\{n_k\}$ then $\lim_{n \to \infty} T_n f(x)$ exists a.e.

Set now

$$T_n^\theta f(x) = \sum_{0 < |k| \leq n} e^{ik\theta} \frac{f(T_k x)}{k}$$

for each $\theta \in [0, 1)$ and $f$ measurable in the dynamical system $(X, \Sigma, m, T)$. 


Problem 3. The fundamental question is whether it is true that there exists a Wiener-Wintner result for these averages, i.e. whether for each $f \in L^\infty(X)$ there exists $X_0 \subset X$ with $m(X_0) = 1$ such that $\lim_{n \to \infty} T_n^\theta f(x)$ exists for each $x \in X_0$ and $\theta \in [0, 1)$.

Problem 4. If one wants to use harmonic analysis to answer this question, one approach is to prove an oscillation inequality for the continuous model and then transfer it back to the ergodic theory setting. This amounts to proving that for each decreasing sequence $\{t_k\}$ of positive real numbers, one has

$$\left\| \sup_{\theta \in [0, 1)} \left( \sum_k \sup_{t_{k+1} \leq t \leq t_k} \int_{|y| \leq t} \frac{f(x) - y}{y} e^{iy\theta} dy \right) \right\|_{2, \infty} \lesssim \|f\|_2,$$

for $f \in L^2(\mathbb{R})$.

Denote also for $f \in L^1([0, 2\pi))$

$$C_n f(x) = \sum_{|j| \leq n} \hat{f}(j) e^{ijx}$$

Problem 5. Is it true that $V_p(C_n, f) : L^2 \to L^2$ boundedly?

H. Furstenberg. A system $(X, \sigma, \tau)$ is a compact metric space $X$ together with two commuting continuous transformations $\sigma, \tau$ acting on $X$. A system $(X, \sigma, \tau)$ satisfies the dimension subadditivity property if for every $x \in X$ we have

$$\dim \{\sigma^m \tau^n x\}_{m, n \in \mathbb{N}} \leq \dim \{\sigma^m x\}_{m \in \mathbb{N}} + \dim \{\tau^n x\}_{n \in \mathbb{N}}$$

where $\dim$ denotes the Hausdorff dimension. It satisfies the transversality property if whenever $A, B \subset X$ are closed such that $\sigma A \subset A, \tau B \subset B$, we have

$$\dim(A \cap B) \leq \max\{\dim A + \dim B - \dim X, 0\}.$$

Problem 6. Let $n \in \mathbb{N}$ and $\sigma, \tau$ be expanding (continuous) homomorphisms of $\mathbb{T}^n$ with the standard metric such that every orbit $\{\sigma^m \tau^n x\}_{m, n \in \mathbb{N}}$ is either finite or dense, $x \in \mathbb{T}^n$. Does the system $(X, \sigma, \tau)$ satisfy the dimension subadditivity and/or the transversality property?

The assumption that both $\sigma$ and $\tau$ are expanding is necessary as one can see by choosing $\sigma = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad \tau = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$; then it can be shown that the system $(\mathbb{T}^2, \sigma, \tau)$ does not satisfy the dimension subadditivity or the transversality property.

Problem 7. Suppose that $\Lambda$ is a finite additive group with the discrete metric. On the sequence space $X = \Lambda^\mathbb{N}$ with the product metric we let $\sigma$ be the shift transformation and $\tau = 1 + \sigma$ (i.e. $(\tau x)_i = x_i + x_{i+1}$). Does the system $(X, \sigma, \tau)$ satisfy the dimension subadditivity property?

Note that this system doesn’t satisfy the transversality property because there are many infinite closed subsets invariant under both transformations.
Let $L$ be the $n \times n$ lower triangular matrix of 1’s, i.e.,

$$L = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 1 & \cdots & \cdots & 1 \end{pmatrix},$$

and $e$ be the $n$ dimensional vector $(1, \cdots, 1)^T$.

Let $x$ be a real number, $\Pi = \{\Omega_1, \ldots, \Omega_K\}$ be a partition of $\mathbb{R}^n$ into Lebesgue measurable sets, and $D = \{d_1, \ldots, d_K\}$ be a set of vectors in $\mathbb{Z}^n$. Given the triple $(x, \Pi, D)$, we define a piecewise affine map $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ by

$$T(u) = Lu + xe + d_i, \text{ if } u \in \Omega_i.$$ 

We say that $T$ is stable if there exists a bounded set $A \subset \mathbb{R}^n$ of positive Lebesgue measure such that $T(A) \subset A$. It is easy to see that $T$ is stable if and only if there exist bounded sets $B$ and $C$ in $\mathbb{R}^n$ of positive measure such that

$$\bigcup_{k=0}^{\infty} T^k(B) \subset C.$$

We would like to understand the invariant sets of the map $T$, given the triple $(x, \Pi, D)$. The following theorem is not difficult to prove:

**Theorem**

Let $x$ be an irrational number and $T$ be stable. Then there exist a finite and non-empty collection of disjoint sets $\tau_1, \ldots, \tau_N \subset \mathbb{R}^n$ such that

(a) each $\tau_i$ tiles $\mathbb{R}^n$ by $\mathbb{Z}^n$ translations, i.e., the collection

$$\{\tau_i + m : m \in \mathbb{Z}^n\}$$

is a partition of $\mathbb{R}^n$, and

(b) if we let $\Gamma = \bigcup \tau_i$, then $T(\Gamma) = \Gamma$. Also, we have $\Gamma = \bigcap_{k=0}^{\infty} T^k(A)$.

**Notes.**

1. Note that the affine map $u \mapsto Lu + xe$ is always unstable in all dimensions. The role of the partition $\Pi$ and the corresponding set of translations $D$ is to overcome this inherent instability.

2. Perhaps the simplest stable maps $T$ are those given (implicitly) by $T(u) = Lu + xe \pmod 1$. Here, we have the invariant set $\Gamma = [0, 1)^n$. (In reality, the corresponding partition $\Pi$ would have infinitely many sets, but one can always reduce it to a finite partition by restricting the $d_i$ to those produced by $u \in [0, 1)^n$. In general, this requires $K = 2^n$.)

3. The most interesting case is when $K = 2$ regardless of the dimension $n$, i.e., when there are only two sets in the partition $\Pi$. This is the most challenging setup for finding stable maps $T$, though it is known that stable maps exist in all dimensions (for values of $x$ in an interval).

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(4) In one dimension, there is a strong link to interval exchange transformations. Higher dimensional versions are not much studied.

Some open problems.

Problem 8. For which \((x, \Pi, D)\) is \(T\) stable?
(Here, it would be desirable to have a characterization, or a set of sufficient or necessary conditions that can be checked, preferably via a finite algorithmic test.)

Problem 9. Determine the (maximal) invariant set \(\Gamma\) as an explicit function of the parameters of \(T\) (i.e., \((x, \Pi, D)\)).

Problem 10. If this is not possible, predict the size of \(\Gamma\) and/or the number of tiles \(N\) in \(\Gamma\). (Here, the single tile case \(N = 1\) is particularly important as then the dynamics of \(T\) within \(\Gamma\) is isomorphic to the skew translation \(u \mapsto Lu + xe \pmod{1}\) on \([0, 1)^n\), which is uniquely ergodic for irrational values of \(x\).)

Problem 11. What is the regularity of \(\Gamma\) for a given map \(T\)?
(For instance, determine the Hausdorff dimension of \(\partial \Gamma\). These results are useful in deriving quantitative results for the convergence of ergodic averages via the standard machinery of Weyl sums.)

Tiling invariant sets are observed even when \(x\) is rational, however the theorem mentioned above is unable to explain this. Explain the tiling phenomenon without using ergodic theory arguments.

J. P. Kahane. [Genericity and Prevalence]

Given a class of functions, if we say that a property holds in general, what does it mean? We restrict ourselves to classes of functions that are Fréchet spaces. A “property” can be identified with a subset of the space.

Since a Fréchet space is a Baire space, we have the notion of generic property, meaning that it holds on a countable intersection of dense open sets. Instead of generic we also say quasi sure.

There is also the notion of almost sure, when we equip the Frechet space with a probability measure (we always assume that it is carried by a \(\sigma\)-compact set in the space).

When the Fréchet space is given, the notion of generic, or quasi sure, is well defined, but the notion of almost sure depends on the choice of a probability.

However, using the group structure of the space, we’ll say that a property is prevalent if it is almost sure for some probability measure and all its translates.

The notion was introduced by Christiansen in 1972, and was developed by Hunt, Sauer, and Yorke in 1992 (Bull. AMS 27, 217–238; see also Bull. AMS 28, 306–307). I learned it from Stéphane Jaffard and his student Aurelia Fraysse, who have a series of examples in various spaces, where the properties they consider (multifractal formalism, failure of analytic continuation) are both generic and prevalent (2005). A simple example in \(C(\mathbb{R})\) is the nowhere-differentiability, known to be generic and also prevalent, using the Wiener measure (Holicky and Zagicek, Acta Univ. Carol 41 (2000), 7–11).

Our general question can be split into two parts:

Problem 12. Give other examples of spaces and properties both generic and prevalent.

Problem 13. Give spaces and properties which are as different as possible from the generic and from the prevalent point of view.

Here is a contribution for question 2): The Fréchet space is the real space \(C(\mathbb{R})\), and we consider a Cantor set \(E\) in \(\mathbb{R}\) and a continuous probability measure \(\mu\) on \(\mathbb{R}\). We look at
properties of the images of $E$ and of $\mu$ by $f \in C(\mathbb{R})$, say, $F = f(E)$ and $\nu = \mu \circ f^{-1}$. Generically, $F$ is a Kronecker set, meaning a Cantor set such that each function continuous on $F$ whose absolute value is 1, can be approximated uniformly by imaginary exponentials. It is a thin set in most aspects of harmonic analysis (R. Kaufman, 1967).

Using this fact, one sees that $\nu$ is generically a singular measure, and moreover

$$\limsup_{n \to \infty} |\hat{\nu}(n)| = 1.$$ 

The prevalent properties are quite different. Prevalently, $F$ has non-empty interior (question: is it a union of intervals?). Prevalently, $\nu$ is absolutely continuous, its density is $C^\infty$ and moreover, given any non quasi-analytic class of infinitely differentiable functions (or a countable intersection of such classes), its density belongs to this class or this intersection.

**Y. Katznelson.** A set $\Lambda \subset \mathbb{N}$ is a set of recurrence for the system $(X, d, T)$ if for every open set $U \subset X$ there exists $n \in \Lambda$ such that $U \cap T^{-n}U$ is nonempty. $\Lambda$ is a set of **topological recurrence** if it is a set of recurrence for every minimal topological system, and a set of **Bohr recurrence** if it is a set of recurrence for every translation on a finite dimensional torus (with the standard metric).

**Problem 14.** If $\Lambda \subset \mathbb{N}$ is a set of Bohr recurrence is it necessarily a set of topological recurrence?

For background see Y. Katznelson, *Chromatic Numbers of Cayley Graphs on $\mathbb{Z}$ and recurrence*, Combinatorica, 21, No 2, 211-219, also available at

http://math.stanford.edu/~katznel

**B. Kra.**

**Problem 15.** Let $(X, B, \mu)$ be a finite measure space and let $T_1, \ldots, T_k$ be ergodic commuting measure preserving transformations acting on $X$. If $\varepsilon > 0$ when is the set

$$\{ n \in \mathbb{N} : \mu(A \cap T_1^{-n}A \cap \ldots \cap T_k^{-n}A) > \mu(A)^{k+1} - \varepsilon \}$$

syndetic?

If $T_1 = T, T_2 = T^2, \ldots, T_k = T^k$, V. Bergelson, B. Host and B. Kra showed that the set is always syndetic for $k \leq 3$ and gave examples where it is empty for $k = 4$.

**Problem 16.** Let $(X, B, \mu)$ be a finite measure space, $T_1, \ldots, T_k$ be commuting measure preserving transformations acting on $X$, and $p_1, \ldots, p_k$ be linearly independent integer valued polynomials with zero constant term. If $\varepsilon > 0$ when is the set

$$\{ n \in \mathbb{N} : \mu(A \cap T_1^{-p_1(n)}A \cap \ldots \cap T_k^{-p_k(n)}A) > \mu(A)^{k+1} - \varepsilon \}$$

syndetic?

For a single transformation (i.e. $T_i = T$ for all $i$) N. Frantzikinakis and B. Kra showed that the set is syndetic for every $k \in \mathbb{N}$.
E. Lesigne. Two measure-preserving dynamical systems \((X, T, \mu), (Y, S, \nu)\) are said to be weakly disjoint if, given \(f \in L_2(X, \mu), g \in L_2(Y, \nu)\), there exist sets \(A \subset X\) and \(B \subset Y\), \(\mu(A) = 1, \nu(B) = 1\), such that for any \(x \in A, y \in B\), the sequence
\[
\frac{1}{N} \sum_{n=0}^{N-1} f(T^nx)g(S^ny)
\]
converges as \(N \to \infty\).

We recall the following properties: disjointness implies weak disjointness; a dynamical system with discrete spectrum is weakly disjoint from any other system; the Chacon map is weakly disjoint from any other system (in particular, from itself); the Morse system is weakly disjoint from any ergodic system; two systems of positive entropy are never weakly disjoint.

**Problem 17.** Assume that a measure-preserving dynamical system is weakly disjoint from any ergodic system. Does it follow that it is disjoint from any measure-preserving dynamical system?

A subset \(E \subset \mathbb{Z}\) is called a set of strong recurrence for a measure-preserving dynamical system \((X, \mu, T)\) if for an arbitrary subset \(A \subset X\) of positive measure, there exists \(\varepsilon > 0\) such that \(\mu(A \cap T^{-n}A) > \varepsilon\) for infinitely many \(n \in E\).

**Problem 18.** Let \(E\) be a set of strong recurrence for an arbitrary ergodic system. Does it follow that \(E\) is a set of strong recurrence for any measure-preserving dynamical system.

A. Naor. Let \(X, Y\) be metric spaces. Assume that there exists a constant \(C\) such that for any subset \(Z \subset X\), any \(K > 0\), and any Lipschitz map \(F : Z \to Y\) with Lipshitz constant \(K\), one can find an extension \(F : X \to Y\), agreeing with \(F\) on \(Z\) and having Lipschitz constant at most \(CK\). The smallest such \(C\) is denoted by \(e(X, Y)\) (we set \(e(X, Y) = \infty\) if no such constant exists).

The classical theorem of Kirszbraun (1934) says that \(e(H_1, H_2) = 1\) for two Hilbert spaces \(H_1, H_2\): in other words, a Lipschitz map from a subset of a Hilbert space into another Hilbert space can be extended to a global map between these spaces without any increase in the Lipschitz constant.

In 1992, Keith Ball showed that \(e(L_2, L_p)\) is finite for \(1 < p < 2\).

**Problem 19.** Is \(e(L_2, L_1)\) finite or infinite?

For maps into Banach spaces, a different approach to the extension problem was suggested by Lee and Naor (2003). Given a metric space \(X\), assume that there exists a constant \(C\) such that for any metric space \(Y\), containing \(X\), any Banach space \(Z\), and any Lipschitz map \(f : X \to Z\) with Lipschitz constant \(K\), there exists an extension \(F : Y \to Z\), agreeing with \(f\) on \(X\) and having Lipschitz constant at most \(CK\). The smallest such \(C\) is called the absolute extendability constant of \(X\) and denoted by \(ae(X)\) (again, we set \(ae(X) = \infty\) if no such constant exists). The problem of estimating the absolute extendability constant is already interesting for finite metric spaces. It is known that there exist two constants \(C_1\) and \(C_2\) such that
\[
C_1 \sqrt{\frac{\log n}{\log \log n}} \leq \sup_{|X| \leq n} ae(X) \leq C_2 \frac{\log n}{\log \log n}.
\]

**Problem 20.** What is the precise asymptotics in \(n\) of \(\sup_{|X| \leq n} ae(X)\)?
D. Ornstein. Find an approach to KAM (i.e. a method to produce invariant curves and surfaces) that is: (i) elementary, (ii) unified, (iii) gives results that are optimal, (iv) relaxes the condition: small perturbation of a completely integrable system. This can be done in dimensions 1 and 2 (work partly joint with Y. Katzenelson).

In dimension 1, given a diffeomorphism $\psi$ of the circle the method gives the complete answer to the smoothness of the invariant measure (i.e. the function conjugating $\psi$ to a rigid rotation) in terms of the smoothness of $\psi$ and the rate of growth of the coefficients of the continued fraction expansion of the rotation number $\alpha$. It gives the optimal results for any diophantine $\alpha$ (completing results of Herman and Yoccoz).

The main results in dimension 2 are:

(A) Let $\psi$ be a measure preserving diffeomorphism of the disc and assume that the rotation number $\alpha$ of $\psi$ restricted to the boundary is diophantine, and that $\psi$ is smooth enough given $\alpha$. Then the method produces invariant curves of optimal smoothness. For example if $\psi \in C^{3+\gamma}$ and the coefficients of the continued fraction expansion of $\alpha$ grow polynomially then there are invariant curves (filling a set of positive measure near the boundary) in $C^{2+\gamma}$. A recent result of Herman produces invariant curves in $C^{1+\gamma}$. He uses standard KAM where the loss in smoothness is not optimal because the curve and its invariant measure are produced together (and the invariant measure is less smooth than the invariant curve). Standard KAM can be applied because if the boundary is rotated rigidly then near the boundary we have a small perturbation of a completely integrable system. Conjugating to this situation requires more smoothness for $\psi$. Note that we could replace the circle by any invariant curve in $C^{1+\gamma}$.

Problem 21. Is there a 3-dimensional analog of (A)?

(B) We assume that $\psi$ is a twist map of the disc or the cylinder. Then a necessary and sufficient condition for the existence of an invariant curve in $C^{\beta}$ ($\beta > 2$) with rotation number $\alpha$ is: A “sufficiently long” finite orbit lies on a $C^{\beta}$ curve. We get the optimal smoothness needed for $\psi$ in terms of $\alpha$ and $\beta$ and “sufficiently long” depends on $\alpha$ and $\beta$. The completely integrable system has disappeared (as in the Morse twist) but the ghost of a “small” perturbation appears in “sufficiently long”. The main machinery of our method does not depend on being in dimension 1 or 2. We can get an analog of (B) for invariant 2-tori in a 3-torus.

Problem 22. Find and prove optimal smoothness conditions in this situation.

Y. Peres. [Projections of planar Cantor sets and a related Kakeya set] Let $K_n$ be the product of two $n$-stage middle-half Cantor sets, and let $K = \cap K_n$ be the product of the two Cantor sets. For $\theta \in [0, \pi)$, let $P_\theta$ be the operator of projection at angle $\theta$. It is well-known (Besicovitch) that

$$\lim_{n \to \infty} \int_0^\pi \mu(P_\theta(K_n))d\theta = \int_0^\pi \mu(P_\theta(K))d\theta = 0.$$ 

Problem 23. What is the rate of convergence? It is known (Peres, Solomyak) that

$$\frac{C}{n} < \int_0^\pi \mu(P_\theta(K_n))d\theta < \frac{c}{\log^*(n)}$$

for some constants $C$ and $c$, where $\log^*(n)$ is defined to be the minimal $k$ such that $n \leq \exp^{(k)}(1)$ and $\exp^{(k)}$ denotes the $k$th iterate of $\exp$.

Note that this is related to a construction of Kakeya set, i.e. a compact set of area zero containing a unit interval in every direction: Let $K$ be a middle half cantor set, and let
$K^{(1)}, K^{(2)} \subseteq \mathbb{R}^2$ be $K^{(1)} = K \times \{0\}$ and $K^{(2)} = \frac{1}{2}K \times \{1\}$. Take $C = \{\alpha x + (1 - \alpha y) | 0 < \alpha < 1, x \in K^{(1)}, y \in K^{(2)}\}$. Then $C$ contains a unit interval in every direction within a range of 60 degrees, and the cuts $C \cap (\{\alpha\} \times \mathbb{R})$ are projections of $K^2$.

**D. Rudolph.** We start by defining an $\mathbb{R}^n$ or $\mathbb{Z}^n$ borel foliation of a Polish space $X$. This will be a countable collection of charts $C_i$ which have the form $S_i \times B_r$, where $S_i$ is Polish and $B_r$, is a ball or box of radius $r$, centered at $\hat{0}$ in $\mathbb{R}^n$ or $\mathbb{Z}^n$. With each chart $C_i$ we have a borel injection $\phi_i : C_i \to X$, i.e.

1. the sets $\phi_i(C_i)$ cover $X$.
2. On intersections $I_{i,j} = \phi^{-1}_i(\phi(C_i) \cap \phi_j(C_j))$ restricted to to a leaf $(s \times B_r) \cap I_{i,j}$ will have the form
   \[ s \times (B_r \cap (f_{i,j,s}(B_r))), \]
   where $f_{i,j,s}$ is an isometry of $\mathbb{R}^n$ or $\mathbb{Z}^n$.
3. for all $x \in X$, $x \in \phi_i(C_i)$ if one seeks to develope the leaf through $x$ starting with $s \times B_r$, $\phi_i(s, \hat{v}) = x$, and extending through leaves of charts that intersect this leaf, one obtains a full copy of $\mathbb{R}^n$ or $\mathbb{Z}^n$.

We call the leaf through $x$, $L_x$. A natural example would be free borel actions of $\mathbb{R}^n$ or $\mathbb{Z}^n$ on a Polish space.

Now suppose $\mu$ is a borel probability measure on $X$. On any chart $C_i$ we can consider $\mu_i = (\phi_i^{-1})_* (\mu)$, the pull back of $\mu$ to a chart. On a chart one can take the Rohklin decomposition of the measure $\mu_i = \int \mu_s d\mu_i(s)$. The measures $\mu_s$ are unique up to normalization. This means that on intersections of charts, the measures $\mu_i$ and $(\phi_i^{-1})_* \mu_j$ agree up to a normalization. This means, for $\mu$ a.e. $x$ one can construct a measure $\mu_x$ obtained by extending the measures chart by chart with the correct normalization, to the full leaf $L_x$. For expliciteness we normalize so that for a.e. $x$, $\mu_x(B_1(x)) = 1$, where $B_1(x)$ is the unit ball about $x$ in $L_x$.

For any $f \in L^1(\mu)$ one can use the $\mu_x$ to compute leaf averages
\[ A_{r,\mu}(x) = \int \frac{f \, d\mu_x}{\mu_x(B_r(x))}. \]

**Problem 24.** Is it true, at this level of generality that $A_{r,\mu}(x)$ converges in $r$ a.s., or in alternatively in $L^1(\mu)$?

It is known that (1) the answer is yes for $\mathbb{R}$ or $\mathbb{Z}$ foliations, by a slight extension of the Hurewicz ergodic theorem. (2) For $\mathbb{R}^n$ or $\mathbb{Z}^n$ foliations one does have a general maximal inequality (these are joint work with E. Lindenstauss).

**W. Schlag.** Let $H_{\omega, \lambda}$ be the discrete quasi-periodic Schrödinger operator given by the skew-shift:
\[ (H_{\omega, \lambda} \psi)_n = -\psi_{n+1} - \psi_{n-1} + \lambda v(T^n_\omega(x, y)) \psi_n, \]
where $(\psi_n) \in l_2$, $v(x, y) = \cos(2\pi x)$ for each $(x, y) \in \mathbb{T}^2$ and $T_\omega(x, y) = (x + y, y + \omega)$ is the skew-shift on the 2 dimensional torus $\mathbb{T}^2$. For each eigenvalue $E$ we denote by
\[ A_j(x, y; E) = \begin{pmatrix} \lambda v(T_\omega^j(x, y)) - E & -1 \\ 1 & 0 \end{pmatrix}, \]
\[ M_n(x, y; E) = \prod_{j=n}^{1} A_j(x, y; E), \]
\[ L_n(E) = \int_{\mathbb{T}^2} \frac{1}{n} \log \|M_n(x, y; E)\| \, dxdy. \]
and \( L(E) = \lim_{n \to \infty} L_n(E) = \inf L_n(E) \) denotes the Lyapunov exponent. Clearly \( L(E) \geq 0 \), for all \( E \) and \( \lambda \). J. Bourgain, M. Goldstein and W. Schlag have proved that for each \( \epsilon \) there exists a set \( \Omega_\epsilon \subset T \) with \( \text{mes}[T \setminus \Omega_\epsilon] < \epsilon \) and a large constant \( \lambda_0(\epsilon) \) such that for each \( \omega \in \Omega_\epsilon \) and \( \lambda \geq \lambda_0 \), the Lyapunov exponents \( L(E) \) are strictly positive for all energies, for a.e. \((x,y)\).

**Problem 25.** Is it true that one can replace \( \lambda \geq \lambda_0 \) with \( \lambda > 0 \) in the above?

This would contrast with the case of simple shift on \( T \), \( T_\omega(x) = x + \omega \), where the result only holds for \( \lambda > 2 \).

Scott Sheffield. [Differentiability of infinity harmonic functions]

We begin with a \( n \times n \)-grid on the plane and a real-valued function \( f(x,y) \), defined on the vertices of the grid.

The game of tug of war has two players and a moving point, which is placed at the origin in the beginning of the game. Each player moves the point into an adjacent vertex of the grid. When the point hits the boundary of the grid at a point \((x_0,y_0)\), the first player collects \( f(x_0,y_0) \) dollars.

**Problem 26.** Is it true that there exist \( N \in \mathbb{N} \) and \( \delta > 0 \) such that if \( n > N \) and \( |f(x,y) - y| < \delta n \), then the optimal first move for the first player is up?

We can now consider the continuous version of the game. Here \( f \) is a continuous function on the boundary of the \( n \times n \)-grid, and the players are allowed to move the point at distance one in any direction. Fix \( C > 0 \). We say that the point goes “up” by a given move of the game if the vertical coordinate of the vector by which it is moved is at least \( C \).

It would be interesting to answer the question in this case also.

B. Solomyak. [Interior points of self-similar sets]

A nonempty compact set \( E \subset \mathbb{R} \) is called self-similar if there exists \( m \in \mathbb{Z}, m \geq 2, \lambda_1, \ldots, \lambda_m \in \mathbb{R}, 0 < \lambda_i < 1, \) and \( d_1, \ldots, d_m \in \mathbb{R} \) such that

\[
E = \bigcup_{i=1}^m f_i(E), \quad \text{where } f_i(x) = \lambda_i x + d_i.
\]

We shall sometimes write

\[
E = E(\lambda_1, \ldots, \lambda_m; d_1, \ldots, d_m).
\]

**Problem 27.** Let \( E \) be a self-similar of positive Lebesgue measure. Does \( E \) have nonempty interior?

If \( m = 2 \), then it is easy to see that the answer is "yes"; we shall therefore assume \( m \geq 3 \) in what follows.

The similarity dimension of \( E \) is \( \alpha > 0 \) such that

\[
\sum_{i=1}^m \lambda_i^\alpha = 1.
\]

It is well-known that the Hausdorff dimension of \( E \) does not exceed its similarity dimension, so we are interested in the case \( \alpha \geq 1 \). If \( \alpha = 1 \), then the answer to Question 27 is "yes" (A. Schief). For \( \alpha > 1 \), it is known (Marstrand and Mattila), that, given \( \lambda_1, \ldots, \lambda_m \), for almost every \( m \)-tuple \( d_1, \ldots, d_m \) the corresponding self-similar set has positive Lebesgue measure. We thus have

**Problem 28.** Take a vector \((\lambda_1, \ldots, \lambda_m)\), whose self-similarity dimension is greater than 1. Is it true that for almost every \((x,y)\) the set \( E = E(\lambda_1, \ldots, \lambda_m; d_1, \ldots, d_m) \) has nonempty interior?
Questions 27 and 28 are already interesting for special families of self-similar sets. For example, let $\frac{1}{4} < \lambda < \frac{1}{2}$, denote by $C_{\lambda}$ the standard middle $1 - 2\lambda$ Cantor set and consider its Cartesian square $K_{\lambda} = C_{\lambda} \times C_{\lambda}$. For $0 \leq \theta \leq 2\pi$, denote by $p_\theta$ the orthogonal projection onto a line with slope $\theta$. Consider the family $p_\theta(K_{\lambda})$. By Marstrand, for almost all $\theta$, the Lebesgue measure of $p_\theta(K_{\lambda})$ is positive. Is it also true that the interior of $p_\theta(K_{\lambda})$ is nonempty for almost all $\theta$? is there at least a single $\theta$ such that $p_\theta(K_{\lambda})$ has positive measure but empty interior?

One may also ask what happens in higher dimensions. Recently, an example was found (joint with M. Csörnyei, T. Jordan, M. Pollicott, and D. Preiss) of a self-similar set in $\mathbb{R}^2$ with positive Lebesgue measure but empty interior.

B. Weiss. [Shannon Entropy of linear factors]

Problem 29. Let $\{X_n\}_{n=-\infty}^{\infty}$ be a stationary process in the reals such that $h(X_n) = \infty$. Assume further that the $X_n$-s are bounded. Let $0 \neq \{c_n\} \in \ell_1(\mathbb{Z})$ and let $Y_n$ be the convolution

$$Y_n = \sum_{k=-\infty}^{\infty} Y_{n-k} c_k.$$ 

Is it true that $h(Y_n) = \infty$?

Problem 30. Suppose $\{X_{i,j}\}_{(i,j) \in \mathbb{Z}^2}$ is a stationary finite valued process. Let $0 \neq \{c_n\} \in \ell_1(\mathbb{Z}^2)$ and let $Y = c * X$. Is it true that $h(X) = h(Y)$?

M. Wierdl. Let $\{\sigma_n\}_{n \in \mathbb{N}}$ be a non-increasing sequence satisfying $0 \leq \sigma_n \leq 1$, and $\{X_n\}_{n \in \mathbb{N}}$ be a sequence of independent $0 - 1$ valued random variables with $P(X_n(\omega) = 1) = \sigma_n$. Given $\omega \in \Omega$ we construct the integer subset $A^\omega$ of $\mathbb{N}$ by taking $k \in A^\omega$ if and only if $X_k(\omega) = 1$. By writing the elements of $A^\omega$ in increasing order we get a sequence $\{a_n(\omega)\}_{n \in \mathbb{N}}$. M. Boshernitzan and J. Bourgain showed that if

$$\lim_{t \to \infty} \frac{w(t)}{\log t} = \infty$$

where $w(t) = \sum_{n \leq t} \sigma_n$ then $\omega$-almost surely we have: in every measure preserving system the "random averages"

$$\frac{1}{w(t)} \sum_{n \leq t} T^n f \in \mathbb{L}^2$$

converge in $L^2$ as $t \to \infty$. Moreover, they showed that $\omega$-almost surely the set $A^\omega$ is a set of (single) recurrence.

Problem 31. Assuming that (1) holds, is it true that $\omega$-almost surely we have: for every measure preserving system and bounded measurable functions $f, g$ the multiple ergodic averages

$$\frac{1}{w(t)} \sum_{n \leq t} T^n f T^{2n} g$$

converge in $L^2$ as $t \to \infty$?

A set $\Lambda \subset \mathbb{N}$ is called a set of double recurrence if for every measure preserving system and measurable set $A$ with positive measure there exists $n \in \Lambda$ such that $\mu(A \cap T^{-n} A \cap T^{-2n} A) > 0$. 

Problem 32. Assuming that (1) holds, is it true that $\omega$-almost surely the set $A^\omega$ is a set of double recurrence?

M. Wierdl notes that the answer to both questions is yes if $\sigma_n = 1/n^a$ for some $0 \leq a < 1/2$, and unknown if $a = 1/2$. 
## Schedule

### Sunday, December 12

<table>
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<tr>
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<tr>
<td>9:15 - 9:45</td>
<td>Coffee and Registration</td>
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<tr>
<td>9:45 - 10:00</td>
<td>Opening remarks</td>
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<tr>
<td>10:00 - 10:45</td>
<td>Jean Bourgain - <em>Harmonic analysis aspects of Ginzburg Landau minimizers</em></td>
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<tr>
<td>11:00 - 11:45</td>
<td>Wilhelm Schlag - <em>On ergodic matrix products and fine properties of the eigenfunctions of the associated difference equations</em></td>
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<tr>
<td>12:00 - 2:00</td>
<td>Lunch break</td>
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<tr>
<td>2:00 - 2:45</td>
<td>Ben Green - <em>Progressions of length four in finite fields</em></td>
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<tr>
<td>3:00 - 3:30</td>
<td>Coffee and refreshments</td>
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<tr>
<td>3:30 - 4:15</td>
<td>Terence Tao - <em>Quantitative ergodic theory</em></td>
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<td>4:30 - 5:20</td>
<td>Open problems session</td>
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<td></td>
<td>• Izzy Katznelson - Topological Recurrence and Bohr Recurrence</td>
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<td>• Mate Wierdl - Multiple convergence and recurrence along random sequences</td>
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<td>• James Campbell - Oscillation and variation estimates for the Carleson operator, and rotated ergodic averages.</td>
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<td>• Daniel Rudolph - Another Ergodic theorem, maybe</td>
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<td>• Idris Assani - On the pointwise convergence of ergodic averages along cubes for not necessarily commuting measure preserving transformations</td>
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<td>• Yuval Peres - Projections of planar Cantor sets and a related Kakeya set</td>
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### Monday, December 13

<table>
<thead>
<tr>
<th>Time</th>
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<tbody>
<tr>
<td>9:30 - 10:15</td>
<td>Mikhail Sodin - <em>Zeroes of Gaussian Analytic Functions</em></td>
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<tr>
<td>10:30 - 11:00</td>
<td>Coffee and refreshments</td>
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<tr>
<td>11:00 - 11:45</td>
<td>Yuval Peres - <em>Determinantal Processes And The IID Gaussian Power Series</em></td>
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<tr>
<td>12:00 - 2:00</td>
<td>Lunch break</td>
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<tr>
<td>2:00 - 2:45</td>
<td>Assaf Naor - <em>The Lipschitz Extension Problem</em></td>
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<tr>
<td>3:00 - 3:30</td>
<td>Coffee and refreshments</td>
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<tr>
<td>3:30 - 4:15</td>
<td>Elon Lindenstrauss - <em>Invariant measures for partially isometric maps</em></td>
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<tr>
<td>4:30 - 5:20</td>
<td>Open problems session</td>
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<tr>
<td></td>
<td>• Scott Sheffield - Infinity harmonic functions</td>
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<td>• Boris Solomyak - Interior points of self-similar sets</td>
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<td>• Emmanuel Lesigne - Two questions around ergodic disintegration</td>
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<td>• Don Ornstein</td>
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<td>• Sinan Gunturk - Invariant sets of a class of piecewise affine maps on Euclidean space and/or other problems</td>
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<td>• Jean-Pierre Kahane - genericity and prevalence</td>
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6:30 - 9:30 Banquet at the Faculty Club in honor of Y. Katznelson, with Bob Osserman as emcee, a historical talk by Jean-Pierre Kahane and a toast by Yonatan Katznelson.

### Tuesday, December 14

<table>
<thead>
<tr>
<th>Time</th>
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<tbody>
<tr>
<td>9:30 - 10:15</td>
<td>Benjamin Weiss - <em>On entropy of stochastic processes</em></td>
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<tr>
<td>10:30 - 11:00</td>
<td>Coffee and refreshments</td>
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<tr>
<td>11:00 - 11:45</td>
<td>Bryna Kra - <em>Lower bounds for multiple ergodic averages</em></td>
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</table>
12:00 - 1:30   Lunch break
1:30 - 2:15   Vitaly Bergelson - IP versus Cesaro
2:30 - 3:15   Hillel Furstenberg - Hausdorff Dimension of Orbit Closures and Transversality of Fractals
3:30 - 4:00   Conference conclusion (Coffee and refreshments)

Abstracts

Vitaly Bergelson - Ohio State University
Title: IP versus Cesaro
Abstract: While traditional ergodic theory concerns itself with the study of the limiting behavior of various Cesaro averages, IP ergodic theory utilizes the notion of IP-convergence which is based on Hindman's finite sums theorem. This usually allows one to refine and enhance the results obtained via the Cesaro averages. An example of such an enhancement is provided by the Furstenberg-Katznelson IP Szemeredi theorem. We shall review some of recent developments in IP ergodic theory and formulate new interesting problems and conjectures.

Jean Bourgain - Institute for Advanced Study
Title: Harmonic analysis aspects of Ginzburg Landau minimizers

Hillel Furstenberg - Hebrew University
Title: Hausdorff Dimension of Orbit Closures and Transversality of Fractals
Abstract: It sometimes happens that the dynamics of a commuting pair of transformations is more easily described than that of the individual transformations. We describe a situation of this type and the possibility that "large" orbits under the two transformations together and a "tiny" orbit under one of the transformations may imply a "large" orbit under the other. Here size is measured by Hausdorff dimension, and we are naturally led to problems regarding fractals.

Ben Green - University of British Columbia
Title: Progressions of length four in finite fields
Abstract: If G is a finite abelian group with size N, define r_4(G) to be the size of the largest subset of G which does not contain four distinct elements in arithmetic progression. Gowers showed that r_4(\Z/N\Z) is bounded above by N/(\log \log N)^c for some c > 0. I would like to discuss some joint work with Terry Tao in which we show that r_4(G) = N/(\log N)^c for the particular group G = (\Z/5\Z)^n. The argument involves `quadratic fourier analysis' on `quadratic submanifolds': I will attempt to explain what that means. I will also discuss the prospects of generalising out result to arbitrary G.

Bryna Kra - Penn State University & Northwestern
Title: Lower bounds for multiple ergodic averages
Abstract: Recent developments in multiple ergodic averages have lead to new combinatorial consequences. I will discuss what happens to a set of integers with positive upper density when it is translated along certain sequences of integers and one takes the intersection of these sets. It turns out that substantially different behavior occurs if the sequence is formed using linear integer polynomials or formed using rationally independent integer polynomials. The first case corresponds to Szemeredi's Theorem for arithmetic progressions, and we have tight lower bounds on the size of the intersection for progressions of length 3 and 4, but no such bounds for longer progressions. For independent polynomials, such as polynomials of differing degrees, we have tight lower bounds on the size of every intersection.

Elon Lindenstrauss - NYU & Clay Mathematics Institute
Title: Invariant measures for partially isometric maps
Abstract: Consider an irreducible non-hyperbolic toral automorphism on the four turus (which is the minimal dimension possible). This map contracts in one (one dimensional) direction, expands in another, and acts isometrically by rotations on the remaining two dimensions. This is prototypical to a larger class of maps which are partially isometric in the same sense. One invariant measure for this map is Lebesgue measure, which as an abstract measure preserving system has been shown by Katznelson to be Bernoulli, just like for the hyperbolic case. However if one consider the class of all invariant measures, there are nontrivial restrictions, and there are many intriguing open questions.

As I will explain in my talk, classifying these invariant measures is closely related to Furstenberg's famous
conjecture about $x^2$ $x^3$ invariant measures on the circle $\mathbb{R}/\mathbb{Z}$.

Part of my talk will be based on joint work with Klaus Schmidt

**Assaf Naor** - Microsoft Research  
**Title:** The Lipschitz Extension Problem  
**Abstract:** The Lipschitz extension problem asks for conditions on a pair of metric spaces $X,Y$ such that every $Y$-valued Lipschitz map on a subset of $X$ can be extended to all of $X$ with only a bounded multiplicative loss in the Lipschitz constant. This problem dates back to the work of Kirszbraun and Whitney in the 1930s, and has been extensively investigated in the past two decades. The methods used in this direction are based on geometric, analytic and probabilistic arguments. In particular, the methods involve stable processes, random projections, random partitions of unity and the analysis of Markov chains in metric spaces. In this talk we will present the main known results on the Lipschitz extension problem, as well as several recent breakthroughs.

**Yuval Peres** - University of California, Berkeley  
**Title:** Determinantal Processes And The IID Gaussian Power Series  
**Abstract:** Discrete and continuous point processes where the joint intensities are determinants arise in Combinatorics (noncolliding paths, random spanning trees) and Physics (Fermions, eigenvalues of Random matrices). For these processes the number of points in a region can be represented as a sum of independent, zero-one valued variables, one for each eigenvalue of the relevant operator. In recent work with B. Virag, we found that for the Gaussian power series with i.i.d. coefficients, the zeros form a determinantal process, governed by the Bergman Kernel. A partition identity of Euler, and a permanent-determinant identity of Borchardt (1855) appear in the proof. The determinantal description yields the exact distribution of the number of zeros in a disk. The process of zeros is invariant for a natural dynamics (we'll see a movie).

**Wilhelm Schlag** - Caltech  
**Title:** On ergodic matrix products and fine properties of the eigenfunctions of the associated difference equations  
**Abstract:** We will present some recent work on difference equations on the one-dimensional lattice. They are typically studied by means of transfer matrices, which define the associated co-cycles. Assuming positive Lyapunov exponents, we will discuss properties of the distribution of eigenvalues in the stochastic limit. This is joint work with Michael Goldstein.

**Misha Sodin** - Tel-Aviv University  
**Title:** Zeros of Gaussian Analytic Functions  
**Abstract:** Geometrically, zeroes of a Gaussian analytic function are intersection points of an analytic curve in a Hilbert space with a randomly chosen hyperplane. Mathematical physics provides another interpretation as a gas of interacting particles. In the last decade, these interpretations influenced progress in understanding statistical patterns in the zeroes of Gaussian analytic functions, and led to the discovery of remarkable canonical models with invariant zero distribution. We shall discuss some of recent results in this area. The talk is based on joint works with Boris Tsirelson.

**Terence Tao** - UCLA  
**Title:** Quantitative ergodic theory  
**Abstract:** There are many techniques used in the study of multiple recurrence (or equivalently in detecting arithmetic progressions and similar objects). The combinatorial and Fourier-analytic approaches tend to work in “finitary” settings such as the cyclic group $\mathbb{Z}/\mathbb{Z}$, whereas the ergodic theory approach works instead in the setting of an infinite measure-preserving system, with the two settings being linked via a transference principle which requires the axiom of choice. The ergodic theory methods are technically simpler (modulo standard machinery such as measure theory and conditional expectation) and are more easily applied to a wide range of problems, but the combinatorial and Fourier methods give more concrete bounds and can apply to certain settings (notably to subsets of sparse pseudorandom sets, of which the primes are a good example) for which there does not yet appear to be an infinitary analogue.

In this talk we discuss a compromise approach, which we dub “quantitative ergodic theory”, in which we work in the finitary setting of $\mathbb{Z}/\mathbb{Z}$ but still exploit the philosophy and ideas from ergodic theory (e.g. sigma algebra factors, almost periodic functions, conditional expectation). This for instance allows one to give an elementary proof of Szemeredi’s theorem based on ergodic methods (requiring no Fourier analysis, and no sophisticated
combinatorial tools other than van der Waerden’s theorem) which also provides a quantitative (but rather poor) bound. This theory was also a crucial component of the recent result of Ben Green and the author that the primes contain arbitrarily long arithmetic progressions.

Benjamin Weiss - Hebrew University of Jerusalem
Title: On entropy of stochastic processes
Abstract: I will discuss some new observations on the relationship between the entropies of two stochastic processes, one of which is a linear factor of the other. The notion of "Finitely Observable" functions of processes will be defined and a remarkable new characterization of the entropy will be given.

Participants

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<tr>
<th>Name</th>
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<td>Tarn Adams</td>
<td>Stanford University</td>
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<td>Andres Angel</td>
<td>Stanford University</td>
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<td>Idris Assani</td>
<td>University of North Carolina, Chapel Hill</td>
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<td>Ioan Bejenaru</td>
<td>University of California, Los Angeles</td>
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<td>Vitaly Bergelson</td>
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<td>Noam Berger</td>
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<td>Alexander Bufetov</td>
<td>Princeton University</td>
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<td>Jean Bourgain</td>
<td>Institute for Advanced Study</td>
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<td>Svetlana Butler</td>
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<td>Ben Green</td>
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<td>John Griesmer</td>
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<td>Raul Lozano</td>
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<td>Mark Meckes</td>
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<td>Roman Muchnik</td>
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Problem: Suppose $(X_n)$ is a bounded stochastic system. Prove that $(X_n)$ is a.m. stationary.