Topics covered in Math 50-series courses

Math 51. Here is a topic list, not arranged by order of coverage or by week.

(1) Vectors, dot products, linear dependence.
(2) Matrices, row reduction, matrix algebra (including inverses and transpose), linear transformations (composition is matrix product, etc.). Introduction to determinants.
(3) Linear equations, null/column space, basis & dimension of subspaces, rank-nullity thm.
(4) Multivariate functions: graphs, level sets, partial derivatives, graphical interpretation of partial derivatives (and tangent planes), higher partial derivatives.
(5) Derivative matrix as linear approximation, Chain rule for multivariate functions, directional derivatives, gradients.
(6) Orthogonal complement, orthogonal projection, least-squares solution to linear systems.
(7) Critical points, extrema, Hessian matrices.
(8) *Brief* introduction to eigenvalues of symmetric matrices, with applications to definiteness and classifying critical points. *No detailed treatment of eigenvectors or characteristic polynomial* (this is done in Math 53).
(9) Lagrange multipliers.

Math 52. Here is a topic list, arranged approximately by week.

(1) Double integrals over various regions.
(2) Double integrals via polar coordinates, and to compute areas and some volumes.
(3) Improper integrals, applications of double integrals, triple integrals.
(4) Cylindrical and spherical coordinates, associated integration formulas, surface area.
(5) Determinants: properties, calculations, geometric meaning (and orientation).
(6) Change of Variables formula for multiple integrals, vector fields and associated derivative operators: div, grad, curl, del.
(7) Line integrals, path independence and Fundamental Theorem for line integrals.
(8) Green's theorems and div, planimeter application, conservative vector fields and curl.
(9) Surface integrals and orientation, relation to flux and tangents.
(10) Divergence Theorem, Stokes' Theorem, conservative vector fields, and applications.

Math 53. Here is a topic list, arranged approximately by week.

(1) Direction fields, equilibrium solutions, first-order linear ODE, separation of variables.
(2) Existence/uniqueness theorem, analysis of equilibria, logistic growth, autonomous equations, Euler's method.
(3) Runge–Kutta, eigenvalues, eigenvectors (including determinants and characteristic polynomials).
(4) Matrix exponential, coupled and uncoupled linear ODE systems, first-order trick, existence/uniqueness for linear systems.
(5) Solving homogeneous systems, Wronskian, complex and repeated eigenvalues, phase-space plots, asymptotic behavior.
(6) Second-degree linear ODE, damped oscillation, non-homogeneous equations, undetermined coefficients.
(7) Forced vibrations (resonance), variation of parameters, Laplace transform.
(8) Inverse Laplace transform, application to solving ODE, review of power series.
(9) Airy equation, Fourier series with examples, Laplace equation.